G_2 gauge theory at finite temperature and density

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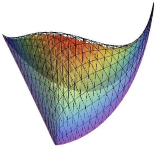
Research Training Group
Quantum and Gravitational Fields



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Polyakov loop in gauge group $\mathcal{G}$

$$P(\vec{x}) = \frac{1}{N_c} \operatorname{tr} \left(\exp \, \mathrm{i} \int_0^{\beta_T} A_0(\tau, \vec{x}) \, d\tau \right), \quad \beta_T = \frac{1}{T}, \quad A_\mu \in \mathfrak{g}$$

- The Polyakov loop expectation value is related to the difference in free energy F due to the presence of an infinitely heavy test quark in the gluonic bath.

$$\langle P \rangle \propto e^{-\beta F}$$

$$\begin{aligned} V_{q\bar{q}}(r) &= -\lim_{T \to 0} T \ln \langle P(\vec{x}) P^*(\vec{y}) \rangle \quad , \quad r = |\vec{x} - \vec{y}| \\ &= c + \sigma r + \alpha r^{-1} + O(r^{-2}) \end{aligned}$$

with string tension σ and Lüscher term $\alpha = -\frac{\pi(d-2)}{24}$

Center transformations for Polyakov loop in representation $\mathcal{R}$:

$$P_{\mathcal{R}} \longrightarrow z^k P_{\mathcal{R}} \quad , \quad z \in \mathbb{Z}(\mathcal{G}) \quad , \quad \text{N-ality } k$$

Confinement means...

...that for every representation $\mathcal{R}$ with non-zero N-ality k $\quad \langle P_{\mathcal{R}} \rangle = 0$

Confinement

$\Longleftrightarrow$

Center symmetry

$\Longleftrightarrow$

nonvanishing asymptotic string tension

In a gauge theory with unbroken non-trivial center symmetry...

- a single static quark cannot exist.
- charges of quarks and anti-quarks cannot be screened by gluons.
- the flux tube (string) between two static charges can never break.
- the Polyakov loop is an order parameter for the confinement-deconfinement phase transition.

If we break center symmetry explicitly (QCD)...

- charges of static quarks and anti-quarks can be screened by dynamical (light) quarks.
- in every representation the string connecting static quarks will break at some distance (there is no asymptotic string tension).
- the Polyakov loop is no longer an order parameter for the deconfinement phase transition.

Confinement is lost

- G_2 is the smallest Lie-group which is simply connected and has a trivial center
- The group has rank 2 and hence possesses two fundamental representations

$$\{7\} \sim \text{quark}, \quad \{14\} \sim \text{gluons}.$$

- Similar as in $SU(3)$ two or three quarks can build a colour singlet

$$\{7\} \otimes \{7\} = \{1\} \oplus \cdots \quad , \quad \{7\} \otimes \{7\} \otimes \{7\} = \{1\} \oplus \cdots$$

- In contrast gluons can screen the colour charge of a single static quark

$$\{7\} \otimes \{14\} \otimes \{14\} \otimes \{14\} = \{1\} \oplus \cdots .$$

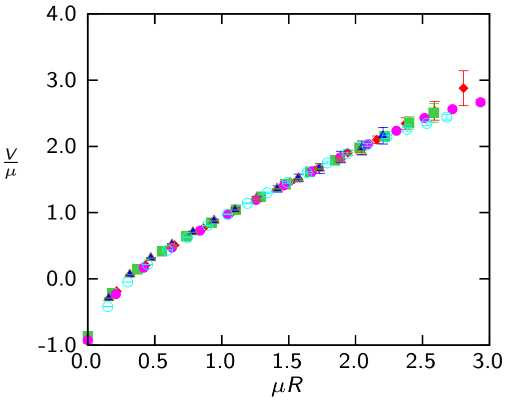
In G_2 gluodynamics...

- every representation $\mathcal{R}$ has zero N-ality
- the Polyakov loop is no longer an order parameter for confinement.
- the flux tube between two static quarks can break due to dynamical gluons.
- there is no linear rising potential up to arbitrary long distances.

Confinement in G_2 gluodynamic really means as in QCD

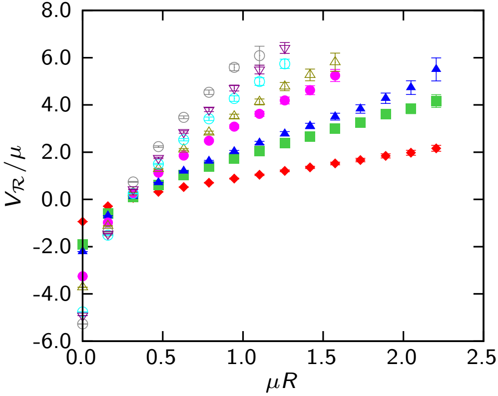
absence of free colour charges in the physical spectrum

linear rising potential only at intermediate scales



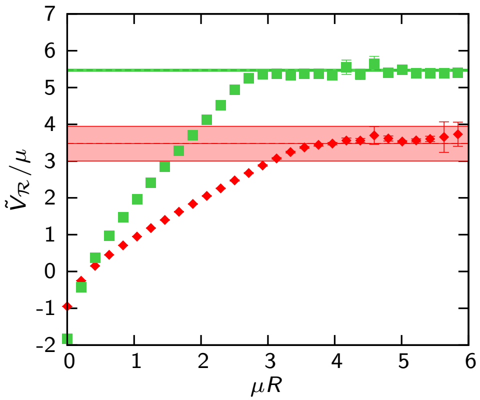
intermediate string tension and Lüscher term

B. Wellegehausen, A. Wipf, C. Wozar, Casimir Scaling and String Breaking in G(2) Gluodynamics, Phys.Rev.D83:016001,2011



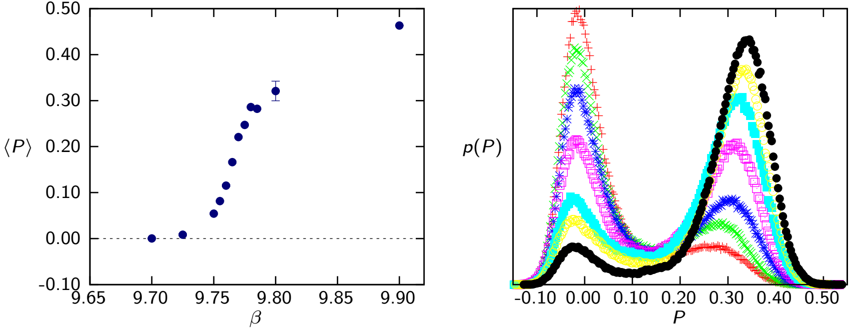
Casimir scaling of string tensions in different representations

B. Wellegehausen, A. Wipf, C. Wozar, Casimir Scaling and String Breaking in G(2) Gluodynamics, Phys.Rev.D83:016001,2011



String breaking even in the fundamental representation at larger distances

B. Wellegehausen, A. Wipf, C. Wozar, Casimir Scaling and String Breaking in G(2) Gluodynamics, Phys.Rev.D83:016001,2011



- The Polyakov loop is an approximate order parameter which changes rapidly at the phase transition and is small (but non-zero) in the confining phase
- First order confinement deconfinement phase transition

K. Holland, P. Minkowski, M. Pepe and U. J. Wiese, Nucl. Phys. **B668** (2003) 207

- Since the center of G_2 is trivial we can study deconfinement without order parameter
- G_2 Yang Mills helps to clarify the relevance of center symmetry for confinement
- Possibility to distinguish between different confinement scenarios
- Similarity to QCD where center symmetry is explicitly broken by matter field

- Interpolation between G_2 and $SU(3)$ gauge theory in G_2 Yang-Mills-Higgs theory
- Possibility to study the G_2-QCD phase diagram at finite temperature and finite chemical potential

The G_2 gauge Higgs model

ϕ: 7-component real scalar field

$$S_{\mathrm{H}}[\phi] = \int d^4x \left(\frac{1}{2}(\partial_\mu \phi, \partial_\mu \phi) + V[\phi] \right) \quad , \quad V[\phi] = \lambda \left(\phi^2 - m^2\right)^2$$

Properties of the model

- invariant under global $SO(7)$
- global $SO(7)$ is spontaneously broken to $SO(6)$ (second order phase transition)
- 7 massive scalar fields $\longrightarrow$ 1 massive and 6 massless scalar fields

Φ: 7-component real scalar field

$$S_{\mathrm{H}}[\Phi] = \frac{1}{2}\kappa \int d^4x \, (\partial_\mu \Phi, \partial_\mu \Phi) \quad , \quad \Phi^2 = 1, \quad \kappa \sim m^2$$

Properties of the model

- invariant under global $SO(7)$
- global $SO(7)$ is spontaneously broken to $SO(6)$ (second order phase transition)
- 7 massive scalar fields $\longrightarrow$ 1 massive and 6 massless scalar fields

$$S_{\text{YMH}}[A,\Phi] = \int d^4x \left(\frac{1}{4g^2}\,\text{tr}\,F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\kappa(D_\mu\Phi, D_\mu\Phi) \right) \quad , \quad \Phi^2 = 1$$

Global $SO(7)$ explicitly broken to local G_2 gauge symmetry

- $SU(3)$ is a subgroup of G_2

$$G_2/SU(3) \sim SO(7)/SO(6)$$

- fundamental representations decompose as

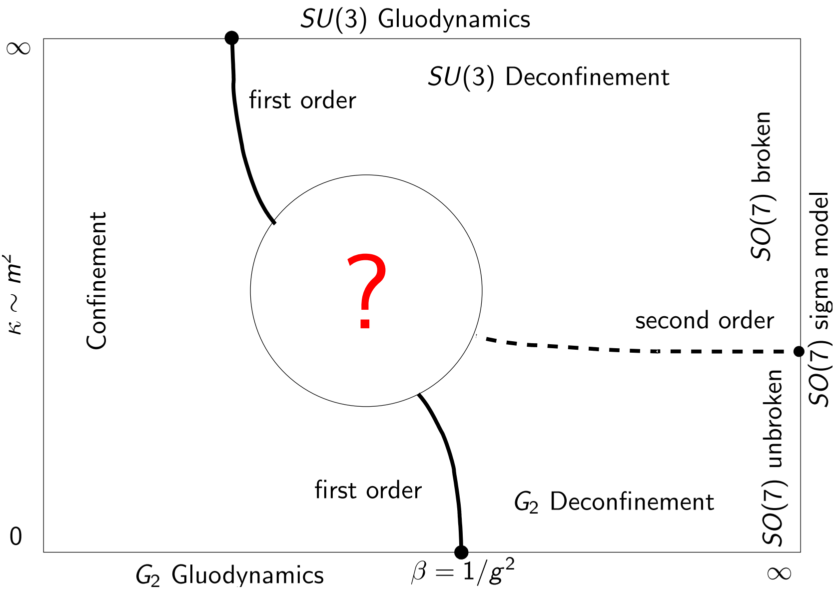
$$\{7\} = \{3\} \oplus \{\bar{3}\} \oplus \{1\} \quad \text{and} \quad \{14\} = \{8\} \oplus \{3\} \oplus \{\bar{3}\}$$

If the Higgs field picks up a vev. . . QCD-like

. . . gluons transforming as $\{3\}$ and $\{\bar{3}\}$ become massive — QCD-Quarks
. . . 8 gluons stay massless — QCD-Gluons

In the limit $\kappa \sim m^2 \to \infty$ . . . $SU(3)$ gluodynamics

. . . massive gluons decouple
. . . $\mathbb{Z}_3$ center symmetry is restored



$SU(3)$ Gluodynamics
∞
first order
$SU(3)$ Deconfinement
$SO(7)$ broken
$SO(7)$ sigma model
$\kappa \sim m^2$
Confinement
?
second order
$SO(7)$ unbroken
first order
G_2 Deconfinement
0
G_2 Gluodynamics
$\beta = 1/g^2$
∞

$$S_{\mathrm{YMH}}[\mathcal{U},\,\Phi] = \beta \sum_{\Box} \left(1 - \frac{1}{N_c}\,\mathrm{tr}\,\mathrm{Re}\,\mathcal{U}_{\Box}\right) + \kappa \sum_{x,\mu} \Phi_x\, \mathcal{U}_{x,\mu}\, \Phi_{x+\mu}$$

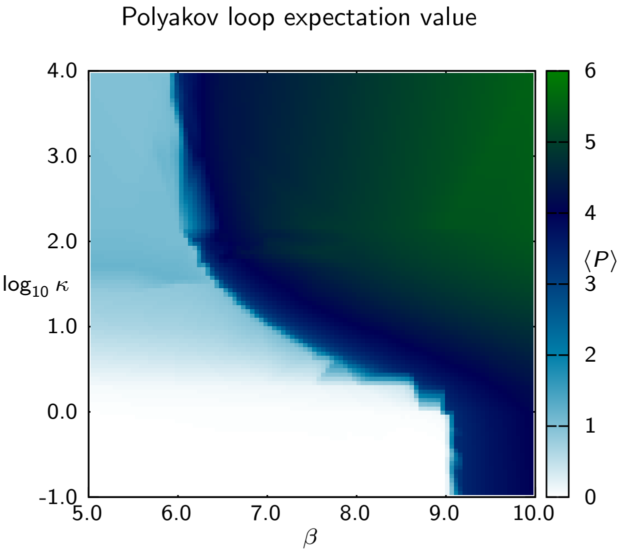
- To investigate G_2 gauge theory we implemented a fast and efficient update algorithm based on a local HMC algorithm.
- We considered finite size scaling of

$$\text{the average plaquette} \quad \mathcal{O}_P = \frac{1}{6\cdot 7\cdot L^3\cdot N_T} \sum_{\Box} \mathrm{tr}\,\mathrm{Re}\,\mathcal{U}_{\Box}\,,$$

$$\text{the Higgs action density} \quad \mathcal{O}_H = \frac{1}{L^3\cdot N_T} \sum_{x,\mu} \Phi_x\, \mathcal{U}_{x,\mu}\, \Phi_{x+\mu}\,,$$

$$\text{the average Polyakov loop} \quad P = \frac{1}{L^3} \sum_{\vec{x}} P(\vec{x})$$

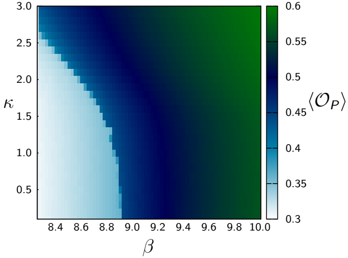
and it's susceptibilities and histogram methods on lattices $6^3 \times 2$ up to $24^3 \times 6$.

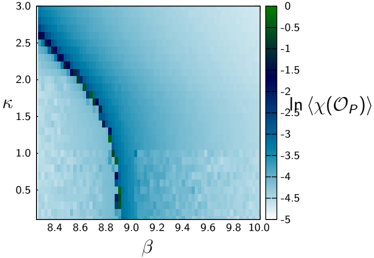


Polyakov loop expectation value
4.0
3.0
2.0
1.0
0.0
-1.0
$\log_{10} \kappa$
5.0
6.0
7.0
8.0
9.0
10.0
β
6
5
4
3
2
1
0
$\langle P \rangle$

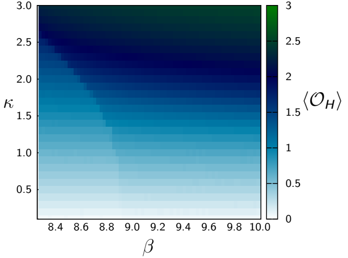
G_2 gauge Higgs model - Phase diagram : Small lattice

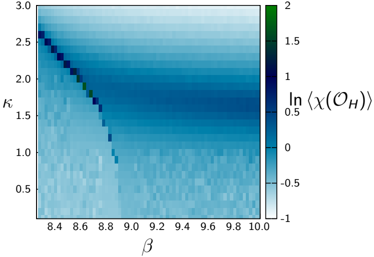
Plaquette density and susceptibility

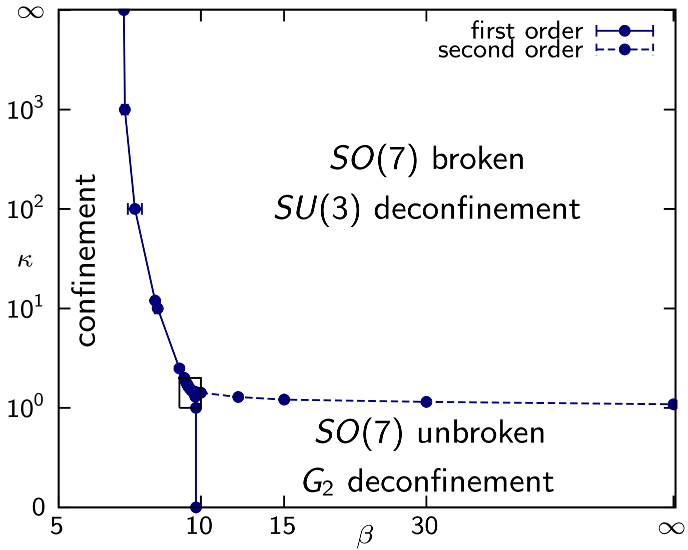




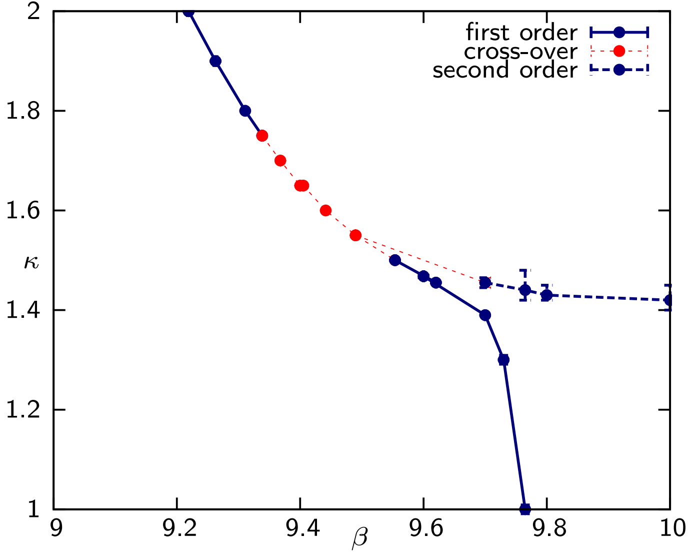
Higgs action density and susceptibility





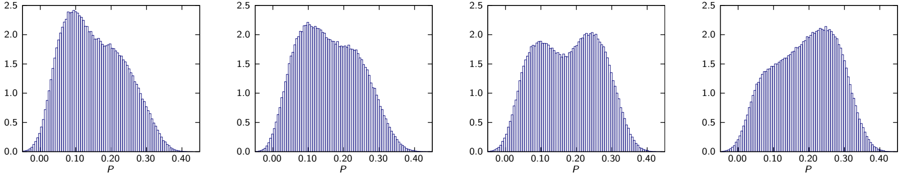


first order
second order
SO(7) broken
SU(3) deconfinement
confinement
SO(7) unbroken
G2 deconfinement
κ
β
∞
10³
10²
10¹
10⁰
0
5
10
15
30
∞

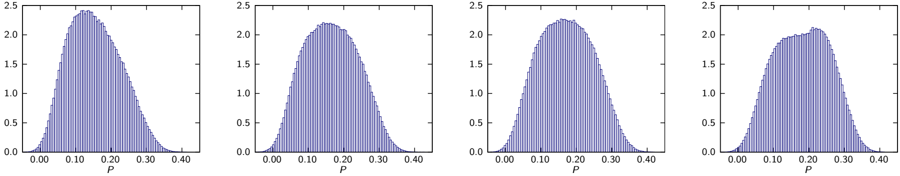


first order
cross-over
second order
2
1.8
1.6
1.4
1.2
1
κ
9
9.2
9.4
9.6
9.8
10
β

Polyakov loop histograms at $\kappa = 1.5$ near the phase transition point ...

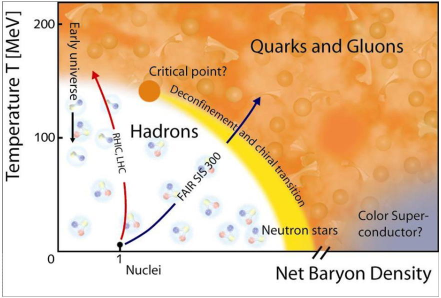


... and at $\kappa = 1.55$



G_2-QCD

- QCD: sign problem $\Longrightarrow$ imaginary chemical potential
- two color QCD: no sign problem, (S. Hands, J.Skullerud, ...)



G_2-QCD

- G_2 is very similar to $SU(3)$
- Fermionic baryons
- No sign problem
- Phase diagram at low temperature and high density

Unitary op. T, $T^\dagger = T^{-1}$ and (lattice) Dirac-Op. $D[\mathcal{U},\mu]$, $\mu \in \mathbb{R}$

$$D^*\, T = T\, D \quad \Longrightarrow \quad \det D \in \mathbb{R}$$

If additionally $T^*T = -\mathbb{1}$, then $\det D \geq 0$

$$D_{xy} = \mathbb{1}\otimes\mathbb{1}(m+dr)\delta_{xy} - \frac{1}{2}\sum_\nu e^{\mu\delta_{\nu 0}}(r-\gamma_\nu)\otimes\mathcal{U}_{x,\nu}\delta_{x+\nu,y} + e^{-\mu\delta_{\nu 0}}(r+\gamma_\nu)\otimes\mathcal{U}^\dagger_{y,\nu}\delta_{x-\nu,y}$$

$$T = F\otimes\Gamma \quad \Longrightarrow \quad F\mathcal{U}F^\dagger = \mathcal{U}^* \quad \text{and} \quad \Gamma\gamma_\mu\Gamma^\dagger = \gamma_\mu^*$$

$\mathcal{U}$ and γ_μ have to be unitary equivalent to real representations.

Euclidean representation for γ-Matrices: $\Gamma = C\gamma_5$ and $\Gamma^*\Gamma = -\mathbb{1}$

For G_2 every representation is real: $F = \mathbb{1} \quad \Longrightarrow \quad T^*T = -\mathbb{1}$

$$\det D[\mathcal{U},\mu] \geq 0$$

- Quarks and anti-quarks are indistinguishable

$$SU(N_f)_L \otimes SU(N_f)_R \otimes U(1)_B \longrightarrow SU(2N_f)_{L=R^*} \otimes \mathbb{Z}(2)_B$$

- We can only distinguish between states with an even or odd number of quarks

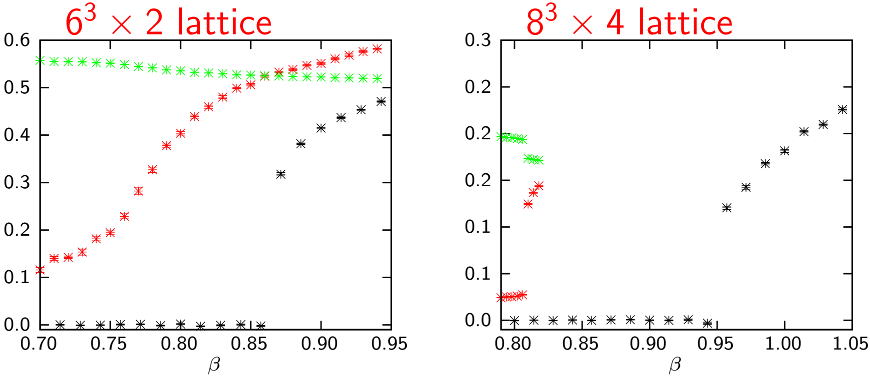
mesons

are even under $\mathbb{Z}(2)_B \quad \Longrightarrow \quad$ bosonic: $qq\,, \quad qqGG\,, \quad \ldots$

baryons

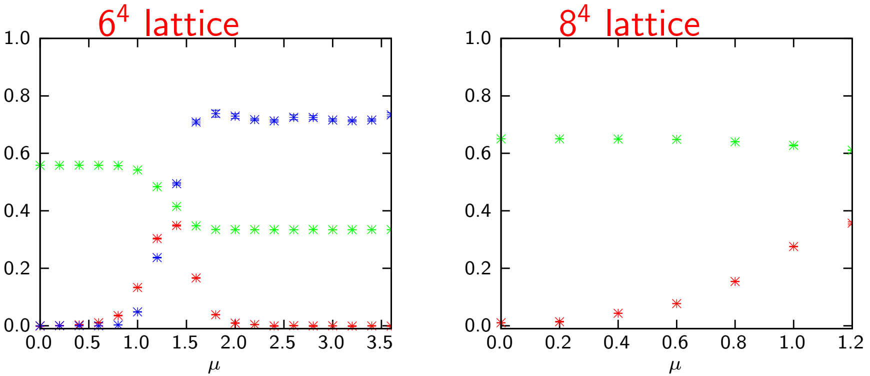
are odd under $\mathbb{Z}(2)_B \quad \Longrightarrow \quad$ fermionic: $qqq\,, \quad qGGG\,, \quad \ldots$

$T \neq 0$ and $\mu = 0$

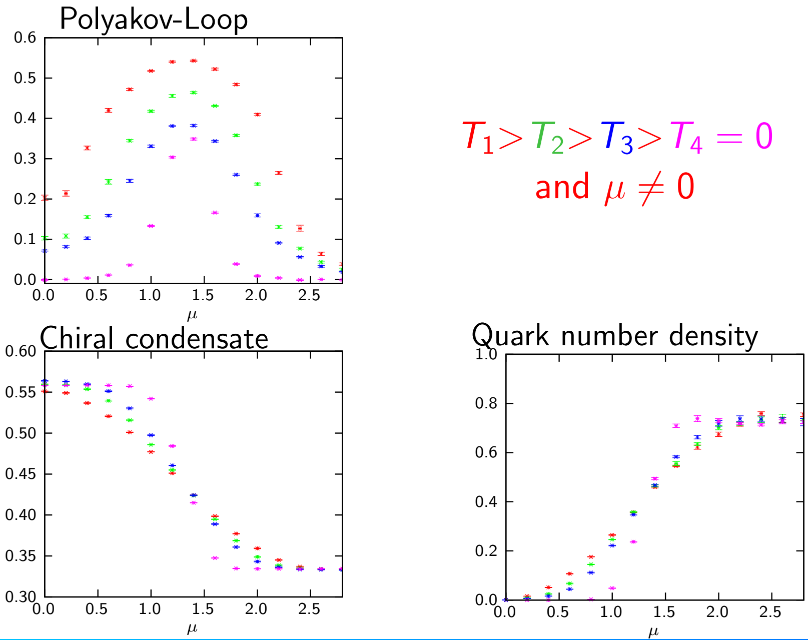


Polyakov-Loop $\langle P \rangle$ Chiral condensate $\langle \bar{\Psi}\Psi \rangle$ quenched Polyakov-Loop $\langle P \rangle$

$T = 0$ and $\mu \neq 0$



Polyakov-Loop $\langle P \rangle$ Chiral condensate $\langle \bar{\Psi}\Psi \rangle$ Quark number density n_q



$T_1 > T_2 > T_3 > T_4 = 0$

and $\mu \neq 0$

- We developed an efficient local hybrid monte carlo algorithm to study gauge theories with different gauge groups, especially G_2, on spacetime lattices.
- We have computed the full phase diagram of the G_2 Gauge-Higgs model in 4 dimensions.
- The confinement-deconfinement transition is first order except for a small window in parameter space where the transition is a crossover or a continuous one.
- Due to the absence of the sign problem we can study G_2-QCD even at low temperature and high density.

B. Wellegehausen, A. Wipf, C. Wozar, Phase diagram of the lattice G_2 Higgs model, Phys. Rev. D 83, 114502 (2011)