Baryon spectroscopy on the lattice

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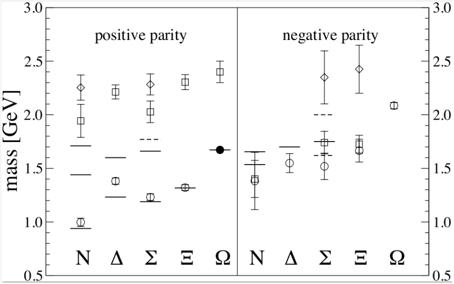
Jena, September 2011

Overview

- **Task**: study the Roper resonance in πN scattering using lattice spectroscopy.
- **Tools**: the variational method and the distillation method represent fundamental tools in the realization of the project.

QCD Spectrum on the lattice

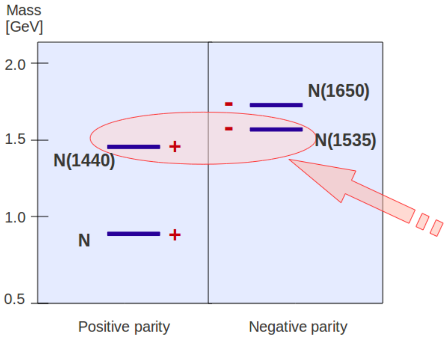


Lattice QCD successfully estimates ground states of hadron spectrum

but

excited states still represent an outstanding challenge.

Roper resonance



$N(1440)P_{11}$ is the lightest excitation of the Nucleon.

A constituent quark model based on $SU(6)$ symmetry predicts the spectrum of the nucleon to be arranged into successive bands of positive and negative parity.

"I spent a much time trying to eliminate the P_{11} resonance"
[L. D. Roper]

Roper resonance

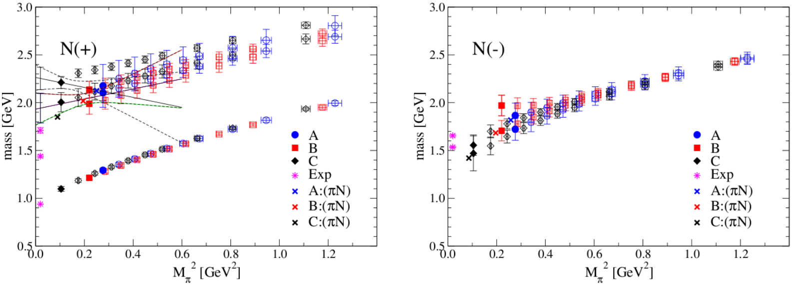
Different models try to explain this parity reversal pattern:

- Flavor-dependent Goldstone boson exchange at low energies.
- Hybrid baryon state large gluonic component q^3G.

The study of the Roper resonance through lattice QCD might shed some light on this open issue.

Roper Resonance on the lattice

Lattice simulations have not been able to reproduce the mass reversal order observed in nature.



Why?

$N\,\pi$ scattering state

- The Roper excitation is compatible with the energy level of the scattering state $\pi\,N$.

It might be necessary to include meson-nucleon interpolators.

- Disconnected diagrams involving backtracking quark loops are needed.

DISTILLATION METHOD

Mass Spectroscopy on the lattice: Ingredients

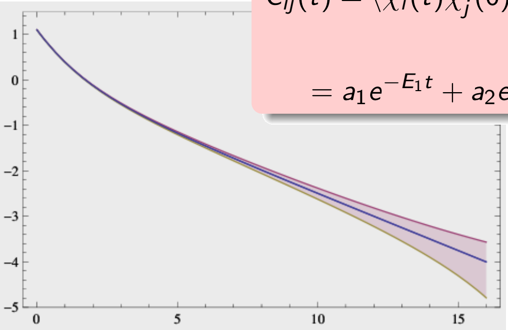
- Action $S = S_{Gauge} + S_{fermion}$
- Gauge configurations with Boltzmann distribution e^{-S}
- Observable for the estimation of the masses of the QCD spectrum: The hadron correlator function.

Hadron Correlation Function

The hadron correlation function in the Euclidean is defined as

$$C_{ij}(t) = \langle \chi_i(t)\chi_j^\dagger(0)\rangle = \sum_n \langle 0|\chi_i|n\rangle e^{-E_n t}\langle n|\chi_j^\dagger|0\rangle =$$

$$= a_1 e^{-E_1 t} + a_2 e^{-E_2 t} + a_3 e^{-E_3 t} + \ldots + \text{noise}$$



Hadron Correlation Function

$$C_{ij}(t) = \langle \chi_i(t) \chi_j^\dagger(0) \rangle = a_1 e^{-E_1 t} + a_2 e^{-E_2 t} + a_3 e^{-E_3 t} + \ldots$$

- exponential decay with the energy
- mass = energy at zero momentum

$$E^2 = m^2 + \mathbf{p}^2$$

- dominated by the ground level

How to extract several energy levels from the correlation function?

Variational method

Consists in disentangling the states using several interpolators.

- Use several interpolators χ_i to construct a basis with minimum overlap.
- Compute the cross correlations $C_{ij}(t) = \langle \chi_i(t) \chi_j^\dagger(0) \rangle$
- Solve the generalized eigenvalue problem

$$C(t) u^{(n)} = \lambda^{(n)} C(t_0) u^{(n)}$$

- Obtain energy levels from the eigenvalues

$$\lim_{t \to \infty} \lambda^{(n)}(t, t_0) = e^{-E_n(t-t_0)}$$

Compute the Correlation Function

What do we need?

- Interpolators
  $\chi_N(y) = \epsilon_{abc}\,\Gamma_1(\alpha,\beta)\Gamma_2(\gamma,\delta)\,u_a(\beta,y)u_b^T(\gamma,y)d_c(\delta,y)$
  $\chi_\pi(y) = u_a(\alpha,y)\,\Gamma(\alpha,\beta)\,\bar{d}_a(\beta,y)$
- Smeared quark sources are needed to optimize the interpolating fields.
- Quark propagator M^{-1}: requires the inversion of a very large matrix.

Determining and storing all the elements of M^{-1} is not possible!

Quark Propagator

- Point-to-all method: compute the propagator for one localized source at given time slice to all the lattice. It works for correlators of single hadron operators concerning connected diagrams.
- We want to study the Roper resonance using multi-hadron operators. Disconnected diagrams are involved.
- To evaluate backtracking loops we need to consider many sources on each time slice: all-to-all propagator.

$N_S^3 \times N_c$ inversions are needed: too expensive!

Solution: Distillation Method

Smeared sources + Cut measurement costs

LapH smearing

$$u(x)\,\Gamma_1\,u(x)\,\Gamma_2\,d(x) \longmapsto S(x,x^{'})u(x^{'})\,\Gamma_1\,S(x,x^{'})u(x^{'})\,\Gamma_2\,S(x,x^{'})d(x^{'})$$

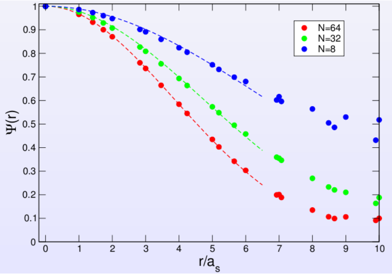
S is the truncated spectral representation in terms of eigenvectors of the 3D Laplacian:

$$S(x,x^{'}) = \sum_{i=1}^{N} c_i\, v_i(x) v_i^{\dagger}(x^{'})$$

Distillation

Distillation defines S on each time slice to be explicitly a very low rank operator:

$$S_t = \square_t(x, x') = \sum_i^{N_v} v_i(x) v_i^\dagger(x') \qquad N_v << (N_S \times N_C)$$



We use a truncated representation of the unit operator

$$\lim_{N_D \to (N_S \times N_C)} \square = 1$$

Distillation

Evaluation of the correlators requires combining various quark lines:

$$Q = S M^{-1} S = \sum_{i,j} v_i (v_i^\dagger M^{-1} v_j) v_j^\dagger$$

compute and store only the elements of

$\sum_{i,j} v_i^\dagger(x) M^{-1} v_j(y)$	instead of	$M^{-1}(x,y)$
$N_v(N_T N_d)$ inversions		$N_S^3 N_c(N_T N_d)$ inversions

Distillation

The Nucleon two point function on each time slice

$$C(t_1 - t_0) = < \mathrm{Tr}\,[\,\Gamma\,\Gamma^\dagger\, M^{-1}_{n,n'}(t_1, t_0)\, M^{-1}_{m,m'}(t_1, t_0)\, M^{-1}_{l,l'}(t_1, t_0)\,] >$$

after the Distillation treatment becomes

$$< \mathrm{Tr}\,[\,\phi_{i,j,k}(t_1)\,\tau_{i,i'}(t_1, t_0)\,\tau_{j,j'}(t_1, t_0)\,\tau_{k,k'}(t_1, t_0)\,\phi^\dagger_{i',j',k'}(t_0)\,] >$$

$$\phi_{i,j,k}(t) = \Gamma\, v_i(t) v_j(t) v_k(t)$$

N_v^3 matrix

$$\tau_{i,i'}(t, t_0) = v_i^\dagger(t) M^{-1}(t, t_0) v_{i'}(t_0)$$

The "perambulator": $(N_d N_v)^2$ matrix

Distillation: Good News

- A considerable amount of computer time is saved: the number of needed inversions is reduced from $N_S^3 N_c$ to a manageable number $N_v = 32, 64, 96$.
- All hadron-hadron correlators can be constructed from the perambulators.
- High flexibility for interpolatos structure.
- Disconnected diagrams become affordable.

Distillation: Bad News

- Many ($N_v \times N_T$) Dirac operator inversions are needed for computing perambulators.
- Volume scaling: to maintain constant resolution $N_v \propto N_S$

Fermion solutions	construct τ	$\mathcal{O}(N_S^2)$
Operator constructions	construct Φ	$\mathcal{O}(N_S^2)$
Meson contractions	$\mathrm{Tr}[\Phi\tau\Phi\tau]$	$\mathcal{O}(N_S^3)$
Baryon contractions	$\bar{B}\tau\tau\tau B$	$\mathcal{O}(N_S^4)$

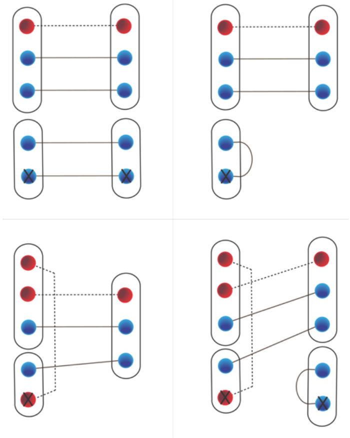
My project: π N scattering

$P+\pi_0 \rightarrow P+\pi_0$

$P+\pi_0 \rightarrow P$

$N+\pi_+ \rightarrow P$

$N+\pi_+ \rightarrow P+\pi_0$



Summary

- The mass reversal order of the Roper resonance is a not yet understood issue and the possibility of reproducing its mass in the lattice framework could be a considerable step forward.
- The attempt of obtaining the right mass pattern on the lattice failed. It might be necessary to investigate the pion-nucleon scattering.
- The distillation method represent a fundamental tool to deal with disconnected diagrams in a reasonable computer time.
- Thanks to the flexibility of the method, the same tool could be used for the study of other baryons of the QCD spectrum.