Phases of Strongly Interacting Matter with Functional Methods

Mario Mitter

PhD-Advisors: Reinhard Alkofer, Bernd-Jochen Schaefer

Karl-Franzens-Universität Graz

Doktoratskolleg Graz "Hadrons in Vacuum, Nuclei and Stars", funded by the Austrian Science Fund: FWF DK W1203-N08.

Jena, September 30, 2011
Table of Contents

1 Motivation

2 Center Symmetry with Fundamentally Charged Scalars

3 Chiral Symmetry with Quarks and Mesons
   - The $N_f$-Flavor Quark-Meson (QM) Model
   - Functional Renormalization Group
   - Results
Motivation
Center Symmetry with Fundamentally Charged Scalars
Chiral Symmetry with Quarks and Mesons

Conjecture for the QCD Phase Diagram

Approaches
- lattice
- functional methods:
  - DSEs
  - FRG
  - nPI-action
- effective theories
- perturbation theory

Order Parameters Accessible with Functional Methods
- chiral transition: quark condensate(s) $\langle \bar{\psi}\psi \rangle$
- center symmetry/confinement transition:
  - “dressed Polyakov loop” (dual chiral condensate) [Gattringer, et al.]
  - dual (scalar) dressing function [Fischer, et al.]
Quantum Field Theory (QFT) with Functional Methods

Generating Functional ↔ n-Point Functions ↔ QFT

- Dyson Schwinger Equations (DSEs):
  - coupled integral equations for \( n \)-point functions
  - from functional integral approach via
    \[
    \int \mathcal{D}\phi \frac{\delta}{\delta \phi} \exp \left[ -S[\phi] \right] = 0
    \]
  - (explicit) regularization/renormalization necessary
Quantum Field Theory (QFT) with Functional Methods

Generating Functional $\leftrightarrow$ $n$-Point Functions $\leftrightarrow$ QFT

- **Dyson Schwinger Equations (DSEs):**
  - coupled integral equations for $n$-point functions
  - from functional integral approach via
  $$\int \mathcal{D}\phi \frac{\delta}{\delta \phi} \exp [-S[\phi]] = 0$$
  - (explicit) regularization/renormalization necessary

- **Functional Renormalization Group (FRG):**
  - functional integral split into momentum shells
  - $\partial_k \rightarrow$ functional differential equation for effective action
  - effective description at scale $k$
  - generates coupled equations for $n$-point functions
Dyson Schwinger Equations
Motivation

Center Symmetry with Fundamentally Charged Scalars
Chiral Symmetry with Quarks and Mesons

QCD

Quark Propagator DSE

\[ \begin{align*}
-1 & = -1 \\
\text{vertex: usually phenomenologically motivated models} & \quad \text{[Maris, Tandy], [Fischer]}
\end{align*} \]

- gluon available from lattice/functional-methods

[Fischer, Maas, Mueller]
QCD

**Quark Propagator DSE**

\[
-1 = -1 -
\]

- gluon available from lattice/functional-methods [Fischer, Maas, Mueller]
- vertex: usually phenomenologically motivated models [Maris, Tandy], [Fischer]

**Quark-Gluon Vertex DSE**

- already in vacuum 12 dressing functions necessary
- scalar part ↔ chiral transition [Alkofer, et al.]
Scalar Propagator DSE

- keep only one-loop terms, no tadpoles
- no Dirac structure: propagator $\propto \frac{Z_S(p^2)}{p^2}$
- order parameter for center symmetry analogous to dual chiral condensate

[Fister, et al.]
**The Center Symmetry Transition with Scalars**

Lattice SU(3) Gluon: Fischer, Mueller, Maas 2010
\[ \mu = 2 \text{ GeV}, \quad \Lambda^2 = 50000, \quad m=0.6 \]

Dual Propagator (zero mode)

[Alkofer, Hopfer, MM, Schaefer in preparation 2011]
Scalar Gluon Vertex

**Vertex DSE at** \( T = 0 \)

- keep only parts that are present in quark-gluon vertex DSE
- no two-scalar-two-gluon vertex contribution (IR-scaling?)
- vertex \( \propto A_p(p^2, q^2, p \cdot q)p^\mu + A_q(p^2, q^2, p \cdot q)q^\mu \)
- main contribution from diagram with three-gluon vertex

![Graphs showing scalar gluon vertex](image)

[Alkofer, Hopfer, MM, Schaefer in preparation 2011]
Functional Renormalization Group

FRG
The Quark-Meson Model (QM-Model)

Mesons

- assumption: quarks/mesons relevant degrees of freedom around $k_X$ ($= \text{scale of spontaneous chiral symmetry breaking}$)
- bound states of quark and antiquark: treated as separate dofs in QM model
- representation of chiral symmetry
- coupled to quarks: Yukawa theory of quarks and mesons
- quark condensates $\leftrightarrow \langle \sigma_{0,3(8)} \rangle$

\[
\mathcal{L}_M = \text{tr} \left[ \partial^\mu \Sigma \partial^\mu \Sigma^\dagger \right] + U (\{\rho_i | i \in \mathbb{N}\}, \xi) - h_b \sigma_b \\
\Sigma = t_b (\sigma_b + i\pi_b) \quad , \quad t_b \text{ generators of } U(N_f) \\
\rho_i = \text{tr} \left[ \left( \Sigma \Sigma^\dagger \right)^i \right], \quad \xi = \text{det} (\Sigma) + \text{det} \left( \Sigma^\dagger \right) \\
\mathcal{L}_{QM} = \mathcal{L}_m + \bar{q} \left( \partial^\mu \gamma^\mu - iht_b (\sigma_b + i\gamma_5 \pi_b) \right) q
\]
Wetterich Equation for Effective Potential

**Derivative Expansion (↔ Low Momentum Expansion)**

- $\Sigma(x) \rightarrow \Sigma$
- only scale-dependency in mesonic potential $U_k$
- three dimensional version of optimized ("Litim") regulator
- flow for $U_k$ with $\Gamma_{k=\Lambda}[\Sigma] = S[\Sigma, 0, 0]$

\[
\partial_k U_k = \frac{k^4}{12\pi^2} \left[ \sum_{b=1}^{2N_f^2} \frac{1}{E_b} \coth \left( \frac{E_b}{2T} \right) \right. \\
- \left. 2N_c \sum_{f=1}^{N_f} \frac{1}{E_f} \left\{ \tanh \left( \frac{E_f + \mu_f}{2T} \right) + \tanh \left( \frac{E_f - \mu_f}{2T} \right) \right\} \right]
\]
$U(2) \times SU(2) \times Z_2$ Quark-Meson Model

with scale-dependent $U_A(1)$ violating 't Hooft determinant coupling
Motivation
Center Symmetry with Fundamentally Charged Scalars
Chiral Symmetry with Quarks and Mesons

The $N_f$-Flavor Quark-Meson (QM) Model
Functional Renormalization Group
Results

Taylor-RG vs. Grid-RG vs. MF

fix explicit breaking term $c_0 = c_{phys}$, Yukawa Coupling $h$ and initial potential $U_{k=\Lambda} = a_{10}(\rho_1 - \rho_{1,0}) + a_{01}(\xi - \xi_0) - c_0\sigma_0 - c_3\sigma_3$
with $m_\pi(k = 0) = 138$ MeV, $f_\pi = 93$ MeV, $m_q = 300$ MeV
$c_3 = ?$, $m_\eta = ?$

- solid line: Taylor-RG
- crosses: grid-RG
- dashed: MF

[Mitter Phases of Strongly Interacting Matter with Functional Methods]

[MM,Schaefer,Strodthoff,von Smekal in preparation 2011]
Spontaneous Breakdown of $Z_2$

- $a_{ij}(k) = (\partial_{\rho_1})^i (\partial_{\xi})^j U_k(\rho_1, \xi) \big|_{\text{min}}$
  - $m_\pi \propto a_{10} + a_{01}$
  - $m_\eta \propto a_{10} - a_{01}$
  - $a_{i(2j)}$ additional symmetry under $\sigma_0 \leftrightarrow \sigma_3$
Spontaneous Breakdown of $Z_2$

- $a_{ij}(k) = (\partial_{\rho_1})^i (\partial_{\xi})^j U_k(\rho_1, \xi)|_{\text{min}}$
  - $m_\pi \propto a_{10} + a_{01}$
  - $m_\eta \propto a_{10} - a_{01}$
  - $a_{i(2j)}$ additional symmetry under $\sigma_0 \leftrightarrow \sigma_3$
  - $U_A(2)$ only broken explicitly by $c_0 \sigma_0 + c_3 \sigma_3$ at $\Lambda$
Motivation Center Symmetry with Fundamentally Charged Scalars
Chiral Symmetry with Quarks and Mesons

The $N_f$-Flavor Quark-Meson (QM) Model
Functional Renormalization Group
Results

Spontaneous Breakdown of $Z_2$

- $a_{ij}(k) = (\partial_{\rho_1})^i (\partial_\xi)^j U_k(\rho_1, \xi)|_{\text{min}}$
- $m_\pi \propto a_{10} + a_{01}$
- $m_\eta \propto a_{10} - a_{01}$
- $a_i(2j)$ additional symmetry under $\sigma_0 \leftrightarrow \sigma_3$
- $U_A(2)$ only broken explicitly by $c_0 \sigma_0 + c_3 \sigma_3$ at $\Lambda$
- solid: $c_0 = c_3 = c_{\text{phys}}$, long dashed: $c_0 = 10c_3 = c_{\text{phys}}$
  short dashed: without quarks

![Graphs showing coupling constants and phases](image)

[MM, Schaefer, Strodthoff, von Smekal in preparation 2011]
\[ c_0 = c_3 = c_{\text{phys}} \]

- additional \( Z_2 \) symmetry \( \sigma_0 \leftrightarrow \sigma_3 \) above chiral \( T_c \)
- \( Z_2 \leftrightarrow \) quark-mass splitting
- calculated order parameter critical exponent: \( \beta = 0.366 \ (0.326) \)

\[ [\text{MM, Schaefer, Strodthoff, von Smekal in preparation 2011}] \]
Restoration of Mesonic Mass Spectrum Degeneracy

- no $U_A(1)$ violating couplings at $k = \Lambda$
- $c_0 = c_{phys}$, $c_3 = 0.1 \times c_{phys}$

[MM, Schaefer, Strodthoff, von Smekal in preparation 2011]
2 + 1 Flavor QM Model

2 + 1 Flavor
Mesonic Masses: FRG vs. Mean-Field (MF)

- Fix initial potential $U_{\Lambda}$ in vacuum to experimental values (physical point).
- Scale independent anomalous mass contribution from 't Hooft determinant.

**Results**

- **Mesonic Masses: FRG vs. Mean-Field (MF)**

  solid line: FRG, dashed line: MF

  MF: Schaefer, Wagner 2009  
  FRG: MM, Schaefer in preparation 2011
fluctuations wash out chiral crossover in light sector

effect on strange sector mitigated

solid line: FRG, dashed line: MF
Motivation
Center Symmetry with Fundamentally Charged Scalars
Chiral Symmetry with Quarks and Mesons

Critical Band

\[ T_{\text{max}}, \mu : \text{maximizes} \ |\nabla \langle \sigma_x(T, \mu) \rangle | \]

\[ I: \frac{|\nabla \langle \sigma_x(T, \mu) \rangle|}{|\nabla \langle \sigma_x(T_{\text{max}}, \mu, \mu) \rangle|} \geq 0.9 \]

\[ \text{II: } \frac{|\langle \sigma_x(T, \mu) \rangle - \langle \sigma_x(T_{\text{max}}, 0, \mu) \rangle|}{|\langle \sigma_x(T_{\text{max}}, 0, \mu) \rangle|} \geq 0.9 \]

\[ \text{III: the same as II with } T_{\text{max}, 0} \rightarrow T_{\text{max}, \mu_c} \]

\[ \chi_{q/\mu^2} \]

Critical band

quark number susceptibility

MF: Schaefer, Wagner 2009
FRG: MM, Schaefer in preparation 2011
Comparison to Two Flavors I

comparison to 2 flavor quark-meson model

MM, Schaefer in preparation 2011
Comparison to Two Flavors II

Comparison to 2 flavor quark-meson model

MM, Schaefer in preparation 2011
Summary and Outlook

- **DSE:**
  - fundamentally charged scalars
  - center symmetry transition and vertex
- **FRG:**
  - $m_{\eta'}$ in $U(2) \times SU(2)$ quark-meson model
  - spontaneous breakdown of $Z_2$
  - phase diagram of $2 + 1$ flavor quark-meson model
Summary and Outlook

- **DSE:**
  - fundamentally charged scalars
  - center symmetry transition and vertex
- **FRG:**
  - $m_{\eta'}$ in $U(2) \times SU(2)$ quark-meson model
  - spontaneous breakdown of $Z_2$
  - phase diagram of 2 + 1 flavor quark-meson model

- **DSE:**
  - scaling solution
  - vertex $@ \ T \neq 0$
- **FRG:**
  - 't Hooft determinant at $\mu > 0$/with $N_f = 3$
  - explore chiral limit in $N_f = 2 + 1$
Influence of $\sigma$-Mass

with anomaly - all other masses fixed
$\leftrightarrow$ no weakening of crossover without anomaly?

MM, Schaefer in preparation 2011
Effect of Taylor-Expansion Order and Cutoff Scale $\Lambda$

with anomaly

![Graph showing the effect of Taylor-expansion order and cutoff scale $\Lambda$.](image)

MM, Schaefer in preparation 2011
Motivation
Center Symmetry with Fundamentally Charged Scalars
Chiral Symmetry with Quarks and Mesons

The $N_f$-Flavor Quark-Meson (QM) Model
Functional Renormalization Group
Results

Mesonic Masses Without Anomaly: FRG vs. MF

... to experimental values (physical point)

![Graph showing mesonic masses vs. temperature for FRG and MF methods.]

solid line: FRG, dashed line: MF

Influence of Anomaly

with anomaly
different strange quark masses

Influence of anomaly

MM, Schaefer in preparation 2011
Sensitivity to Variation of $h_{0,3}$

- **solid line**: $h_0 = h_3 = h_{phys}$
- **short dashed**: $h_0 = h_{phys}$, $h_3 = 0.1 \times h_{phys}$
- **long dashed**: $h_0 = h_3 = 0.1 \times h_{phys}$

![Graphs showing masses and condensates vs. temperature](image)

[MM, Schaefer, Strodthoff, von Smekal in preparation 2011]
Taylor vs. Grid Revisited

\[ \sigma^2 \text{ couplings at origin [MeV}^2] \]

\[ \sigma^2 \text{ couplings at minimum [MeV}^2] \]

[MM, Schaefer, Strodthoff, von Smekal in preparation 2011]