Renormalization group flow for supersymmetric O(N) models

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in collaboration with
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Overview

Scaling behaviour

$S \equiv \Gamma_{k\to\Lambda}$

Γ_k

$\Gamma \equiv \Gamma_{k\to 0}$

Masses of excitations

$\mathcal{N} = 1$ supersymm. $O(N)$ model in $d = 3$

Phase structure of effective theory

(critical behaviour, critical exponents, preserved & broken symmetries, $\cdots$)

Identification of physically relevant parameters

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The $\mathcal{N} = 1$ SUSY $O(N)$ model in $d = 3$

- $O(N)$ symmetry (continuous, global, internal)
- Supersymmetry (continuous, global, external)
- superfield Φ:

$$(x, \theta) \in \mathbb{R}^{3|2} \longmapsto \Phi(x, \theta) \in \mathcal{M} = \mathbb{R}^N \text{ (flat)}$$

- coordinates on $\mathcal{M}$:

$$\left\{\Phi^i(x, \theta)\right\}, \quad i = 1, ..., N = \dim(\mathcal{M})$$

- representation of Φ^i:

$$\Phi^i(x, \theta) = n^i(x) + \bar{\theta}\Psi^i(x) + \frac{1}{2}\bar{\theta}\theta F^i(x)$$

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- action:

$$S[\Phi^i] = \int d^3x \, d\theta \, d\bar{\theta} \left[-\frac{1}{2}\Phi^i \bar{\mathcal{D}}\mathcal{D}\Phi_i + 2NW\left(\frac{\mathcal{R}}{N}\right) \right];$$

$$\mathcal{R} = \frac{1}{2}\Phi^i\Phi_i; \quad \mathcal{D} = \partial_\theta + i\partial\!\!\!/\theta$$

- SUSY: $\delta_\epsilon S = i\bar{\epsilon}\mathcal{Q}S = 0; \quad \mathcal{Q} = -i\partial_{\bar{\theta}} - \partial\!\!\!/\theta$

- action in components (on-shell):

$$S = \int d^3x \left[-\frac{1}{2}n\partial^2 n - \frac{i}{2}\bar{\Psi}\partial\!\!\!/\Psi - \frac{1}{2}W'\bar{\Psi}\Psi - \frac{1}{2N}W''(\bar{\Psi}n)(\Psi n) - \bar{\rho}W'^2 \right]$$

$\Rightarrow$ *potential* $V(\bar{\rho}) = \bar{\rho}\,W'^2\left(\frac{\bar{\rho}}{N}\right), \quad \bar{\rho} := \frac{1}{2}n^i n_i$

$\Rightarrow E_0 = 0 \quad \Rightarrow$ supersymmetry unbroken

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The method: ERGE

effective average action Γ_k

microscopic action
$\Gamma_{k\to\Lambda} \equiv S$

transition:

determined by
flow equation
Wetterich, '93

effective action
$\Gamma_{k\to 0} \equiv \Gamma$

$$\partial_k \Gamma_k = \frac{\mathrm{i}}{2} \mathbf{S}\mathrm{Tr}\left[\partial_k R_k \left(\Gamma_k^{(2)} + R_k\right)^{-1}\right]$$

Synatschke-Czerwonka, Braun, Wipf, '10

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- *truncation*: **LPA** (gradient expansion)
  $\Gamma_k[\Phi^j] = \int d^3x d\theta d\bar{\theta} \left[-\frac{1}{2}\Phi^i \bar{\mathcal{D}}\mathcal{D}\Phi_i + 2NW_k(\frac{\mathcal{R}}{N}) \right]$

- choose adequate regulator functional $\Delta S_k[\Phi^j]$

- $\Rightarrow$ flow equation for W_k:
  $\partial_k W_k(\frac{\bar{\rho}}{N}) = -\frac{k^2}{8\pi^2}\left[\left(1 - \frac{1}{N}\right)\frac{W_k'}{k^2 + W_k'^2} + \frac{1}{N}\frac{W_k' + 2(\bar{\rho}/N)W_k''}{k^2 + (W_k' + 2(\bar{\rho}/N)W_k'')^2} \right]$

  $\Rightarrow$ *nonlinear 2nd order PDE*

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  $\Rightarrow$ *consider* ***large-N limit*** *(LPA exact)*

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1. Evolution of potential?
2. Scaling solution & critical exponents?
3. Spontaneous breaking of scale invariance?

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- **Evolution of potential?**

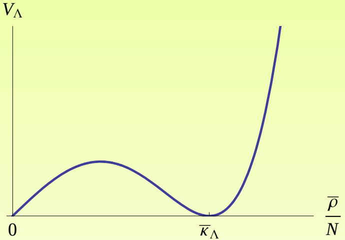
task

PDE: $\partial_k W_k'(\frac{\bar\rho}{N}) = -\frac{k^2}{8\pi^2} W_k'' \frac{k^2 - W_k'^2}{(k^2+W_k'^2)^2}$

IC: $W_{k\to\Lambda}(\frac{\bar\rho}{N}) = \frac{\bar\lambda_\Lambda}{2}(\frac{\bar\rho}{N} - \bar\kappa_\Lambda)^2$

Bardeen, Higashijima, Moshe, '85
Dawson et al., '06

$$\Rightarrow V_\Lambda(\bar\rho) = \bar\rho\,(W_\Lambda')^2 = \bar\lambda_\Lambda^2 \bar\rho\,(\frac{\bar\rho}{N} - \bar\kappa_\Lambda)^2$$



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solution

$$\bar\rho - \bar\rho_0 = \frac{N}{8\pi^2} W_k' \left[\frac{kW_k'}{k^2+W_k'^2} - \frac{\Lambda W_k'}{\Lambda^2+W_k'^2} + 2\arctan(\frac{W_k'}{k}) - 2\arctan(\frac{W_k'}{\Lambda}) + \frac{1}{\lambda_\Lambda}\right]$$

$$\bar\rho_0(k) = \frac{N}{8\pi^2}(k + \delta\kappa_\Lambda \Lambda), \qquad \delta\kappa_\Lambda := \kappa_\Lambda - 1$$

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- **Evolution of potential?**

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PDE: $\partial_k W'_k\left(\frac{\bar\rho}{N}\right) = -\frac{k^2}{8\pi^2} W''_k \frac{k^2 - W'^2_k}{(k^2 + W'^2_k)^2}$

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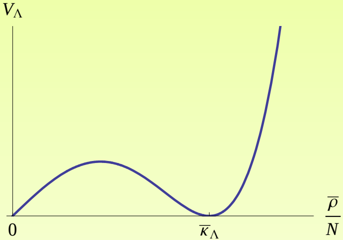

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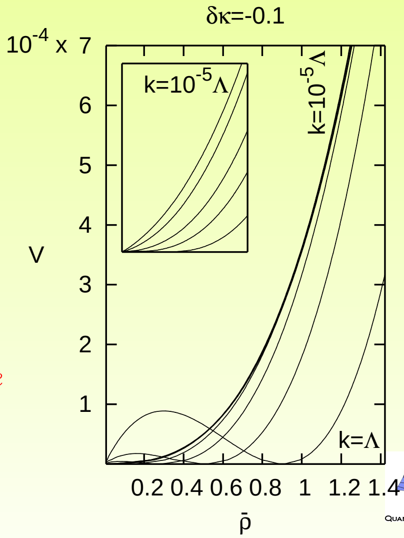
- **Evolution of potential?**

(1) $\delta\kappa_\Lambda < 0$

(2) $\delta\kappa_\Lambda = 0$

(3) $\delta\kappa_\Lambda > 0$

$\Rightarrow$ *$O(N)$ symmetric phase*



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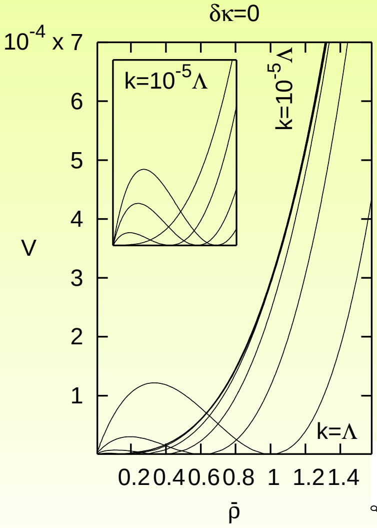
$\Rightarrow$ *phase transition*



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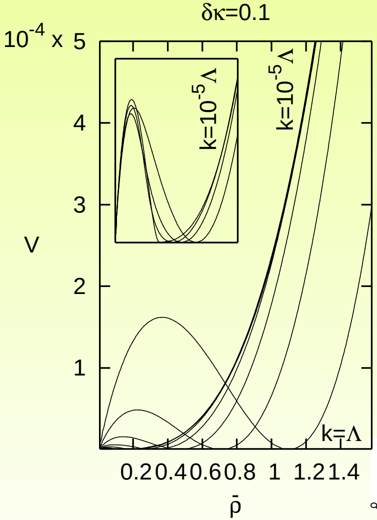

$\Rightarrow O(N)$ *symmetry may be spont. broken*

Bardeen, Higashijima, Moshe '85

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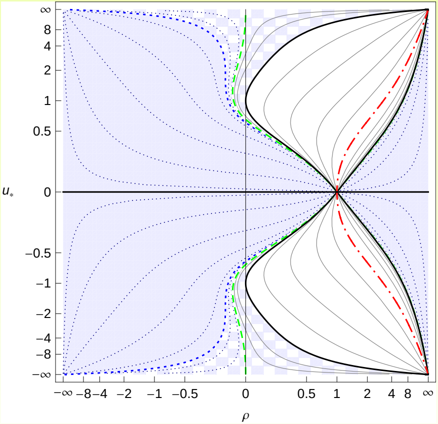
- **Scaling solution & critical exponents?**
- **IR scaling solution** $u_* \Leftrightarrow \kappa_\Lambda = 1$ & $\lambda_\Lambda = c^{-1}$
- line of non-Gaussian fixed points with *Gaussian critical exponents* $\theta = 1, 0, -1, -2, -3, \cdots$

Litim, Mastaler, Synatschke-Czerwonka, Wipf '11



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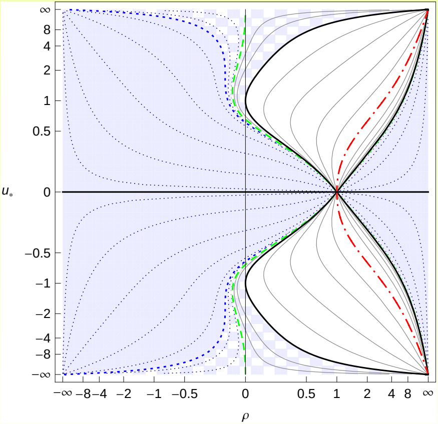

- **Spontaneous breaking of scale invariance?**
- scaling solution ⇒ dynamical *generation of mass* $\bar{m} \neq 0$?
- $\bar{m}^2 = V''(n)|_{n=0} = V'(\bar{\rho})|_{\bar{\rho}=0} = u_*^2(0) \cdot k^2$
- scaling solution: $-1 - \frac{u_*^2}{1+u_*^2} - 2u_* \arctan(u_*) = \frac{1}{\lambda_\Lambda} u_*$

$$\Rightarrow \pi u_* + \mathcal{O}(u_*^{-2}) = \frac{1}{\lambda_\Lambda} u_*$$

$\Rightarrow$ *observe arbitrary mass* $\Leftrightarrow \quad \lambda_\Lambda^{-1} = \pi$

Bardeen, Higashijima, Moshe '85

but: - physical solution?
- finite N: only unique solutions

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Summary

- supersymmetry on all scales *preserved*
- relevant coupling κ_Λ determines *macrophysical behaviour* of theory; *fine tuning* in UV leads to scaling solution
- find line of nontrivial fixed points, classified by exactly marginal coupling λ_Λ and with *Gaussian critical exponents*
- spont. breaking of scale invariance = large-N *artifact*

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