Symmetry and Discretisation in the O(3) Supersymmetric NLSM

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Supersymmetry

- powerful mathematical concept
- provides WIMP's for cosmology
- important ingredient in generalizations of the Standard Model, e.g. the Minimal Supersymmetric Standard Model (MSSM)

MSSM ...

- is formulated as four-dimensional Super-Yang-Mills theory.
- takes considerable effort to study.

Supersymmetric **N**on**L**inear **S**igma **M**odel ...

- is a two-dimensional QFT without gauge degrees of freedom.
- shares some important properties with MSSM.
- can be tackled by less effort.

SUSY O(N) Sigma Model

- introduced 1977 by E. Witten and parallel by P. Di Vecchia and S. Ferrara [Wit77, VF77]

(Euclidean) Continuum Action

$$S = \frac{1}{2g^2} \int d^2x \left(\partial^\mu \mathbf{n} \partial_\mu \mathbf{n} - i\bar{\psi}\partial\!\!\!/\psi + \frac{1}{4}\left(\bar{\psi}\psi\right)^2 \right)$$

(Euclidean) Path Integral

$$Z = \int \mathcal{D}\mathbf{n}\mathcal{D}\psi \; \delta(\mathbf{n}^2 - 1)\delta(\mathbf{n}\psi) \; e^{-S[\mathbf{n},\psi]}$$

- three major symmetries at the classical level: O(N) symmetry, (discrete) chiral symmetry and Supersymmetry

Analytical Results

- $1/N$-expansion by Alvarez shows spontaneous chiral symmetry breaking and nonzero masses [Alv78]
- R. Shankar and E. Witten construct the exact S-Matrix for $N > 4$ and find supermultipletts [SW78]
- J. Evans and T. Hollowood confirm nonzero masses for arbitrary $N > 2$ [EH91]

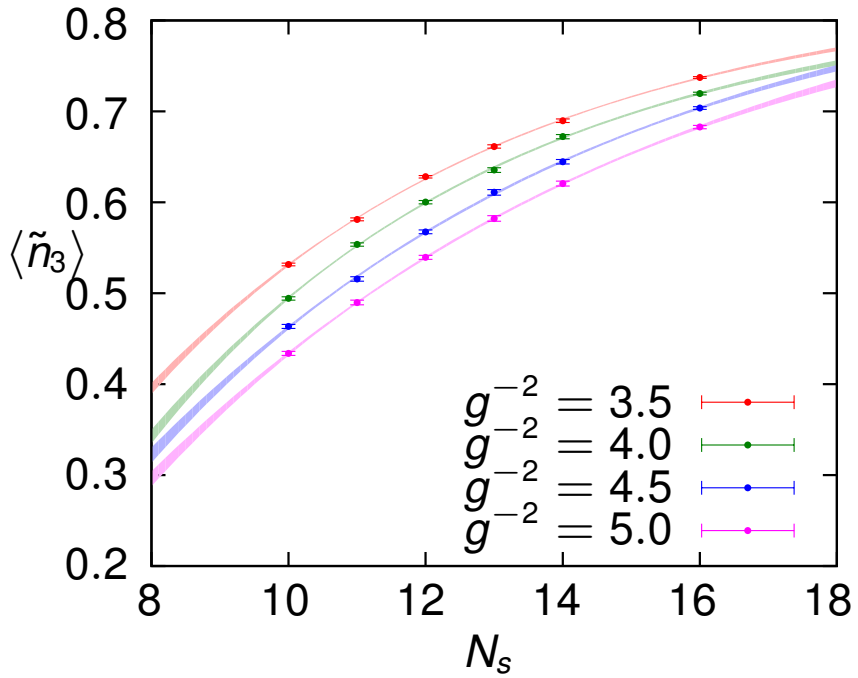
Important Properties

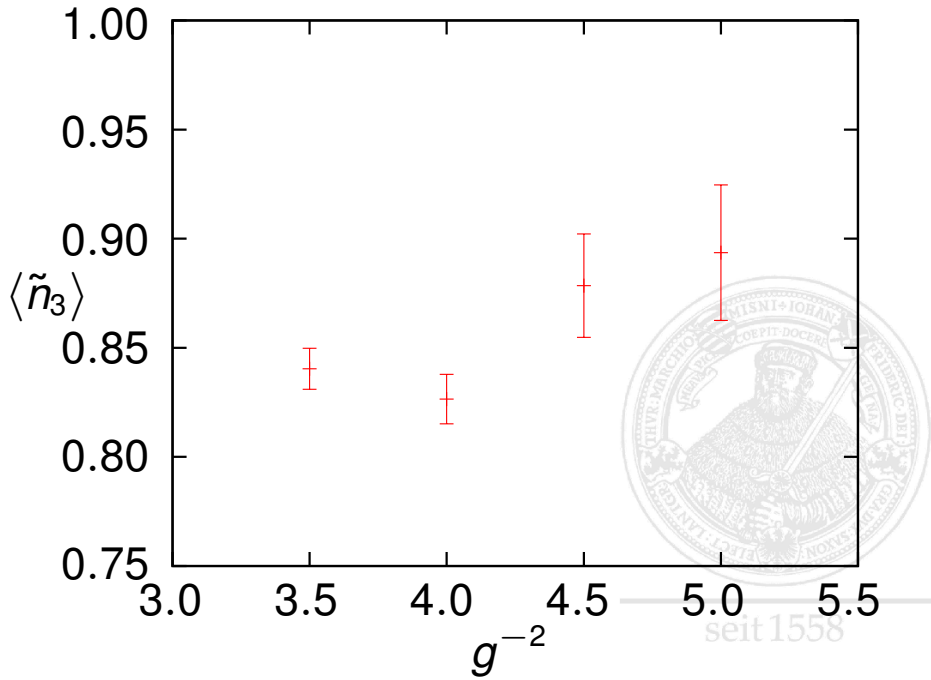
- dynamically generated masses
- asymptotic freedom
- instanton solutions (only for $N = 3$)
- extended $\mathcal{N}{=}2$ supersymmetry algebra (only for $N = 3$)

Failure of Discretization

Previous Lattice Study

- S. Catterall and S. Ghadab [CG04, CG06] utilize extended supersymmetry algebra to construct nilpotent charge
- supersymmetric continuum limit without finetuning, but: O(3) symmetry NOT restored in the continuum limit!





Technical Difficulties

Our Approach: Exact O(3) Symmetry on the Lattice

- stereographic projection $n_{0/1} = \frac{2u_{0/1}}{1+\mathbf{u}^2}$, $n_2 = \frac{1-\mathbf{u}^2}{1+\mathbf{u}^2}$
- group-valued fields $\mathbf{n}_x = O_x\mathbf{c}$, $O_x \in O(3)$
- solves all constraints and allows numerical investigation
- supersymmetric continuum limit must be determined by simulations

Fermionic Derivative: Wilson

- fast (ultralocal)
- explicit breaking of chiral symmetry
- may lead to non-susy continuum limit; alternative?

SLAC Derivative

- introduced 1976 by Drell, Weinstein and Yankielowicz [DWY76]
- reproduces continuum dispersion relation on the lattice for free fermions

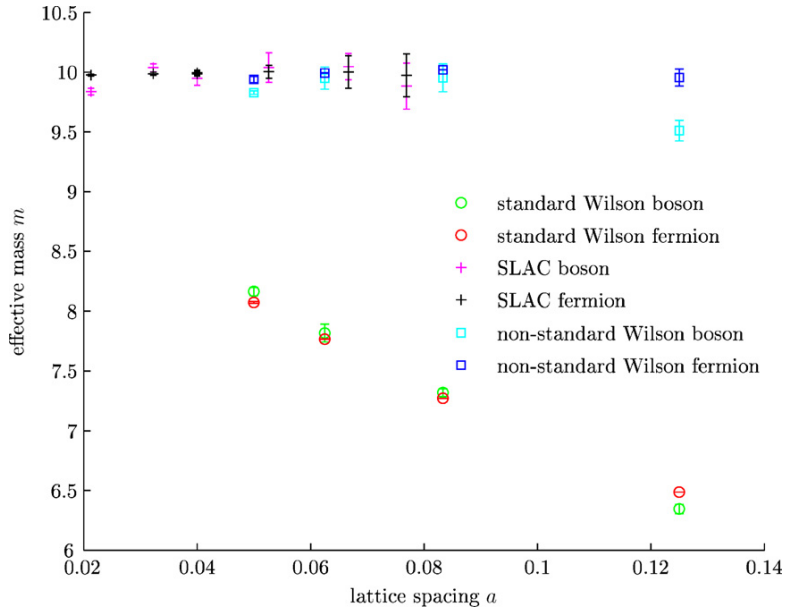
Periodic BC (N odd)

$$\partial_{xy}^{SLAC} = \begin{cases} 0 & : x = y \\ \frac{\pi}{aN}(-1)^{x-y} \sin^{-1} \frac{\pi(x-y)}{aN} & : x \neq y \end{cases}$$

Nielsen-Ninomiya-Budget

- preserves chiral symmetry ✓
- doublerfree ✓
- local ×

SLAC Derivative II



cf. [BKUW08]

Scalar Models

- SLAC shows continuum behaviour even at finite lattice spacing in two-dimensional Wess-Zumino models [BKUW08]

Gauge Theories

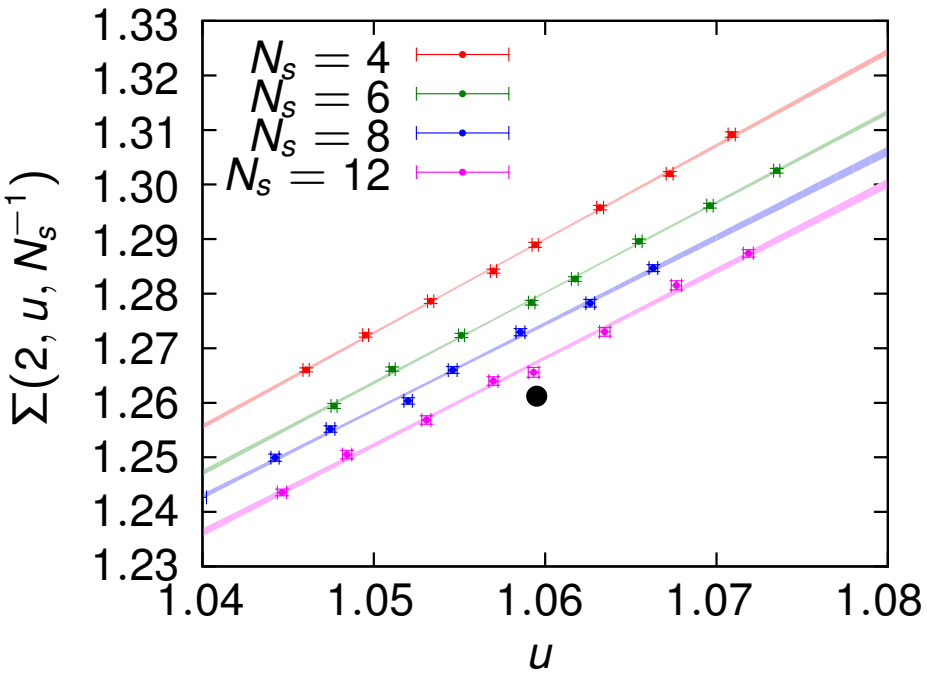
- non-local and non-covariant continuum limit in lattice QED [KS79]
- SLAC derivative suitable for models with curved target space?

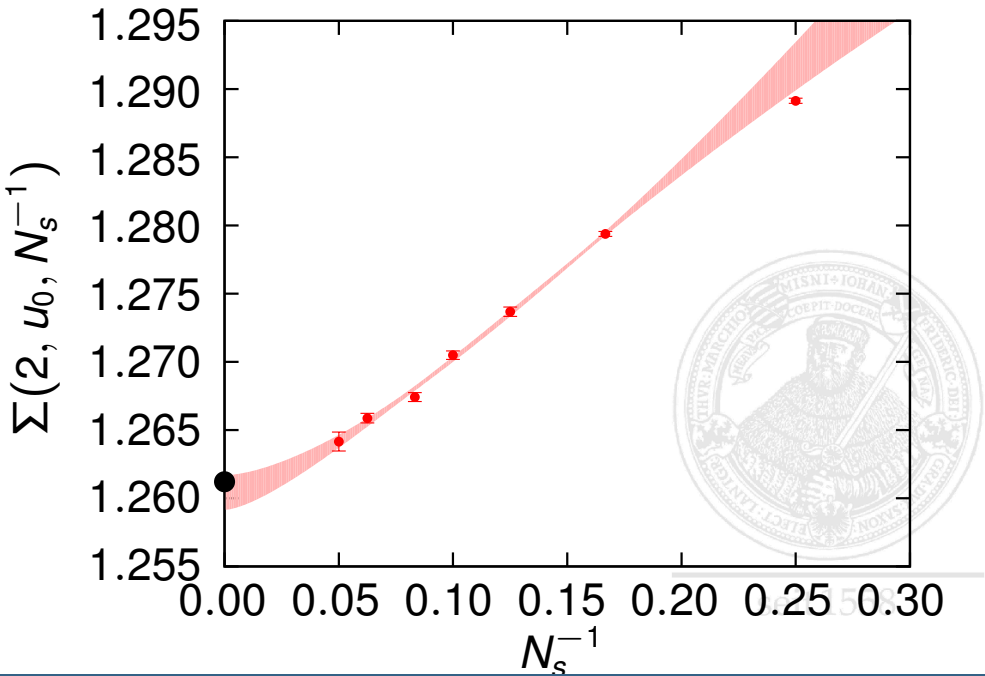
Step-Scaling Function

- scaling $\Sigma(2, u, N_s^{-1})$ of the finite volume mass gap expected to have universal continuum limit $\sigma(2, u) = \lim_{N_s \to \infty} \Sigma(2, u, N_s^{-1})$

$$\Sigma(2, u, N_s^{-1}) = m(2N_s)2L\big|_{m(N_s)L=u}$$

- systematic errors: excited states, finite N_t, $O(a^2(\log a)^3)$ artifacts





Step-Scaling Function II

Apparent O(a) Lattice Artifacts

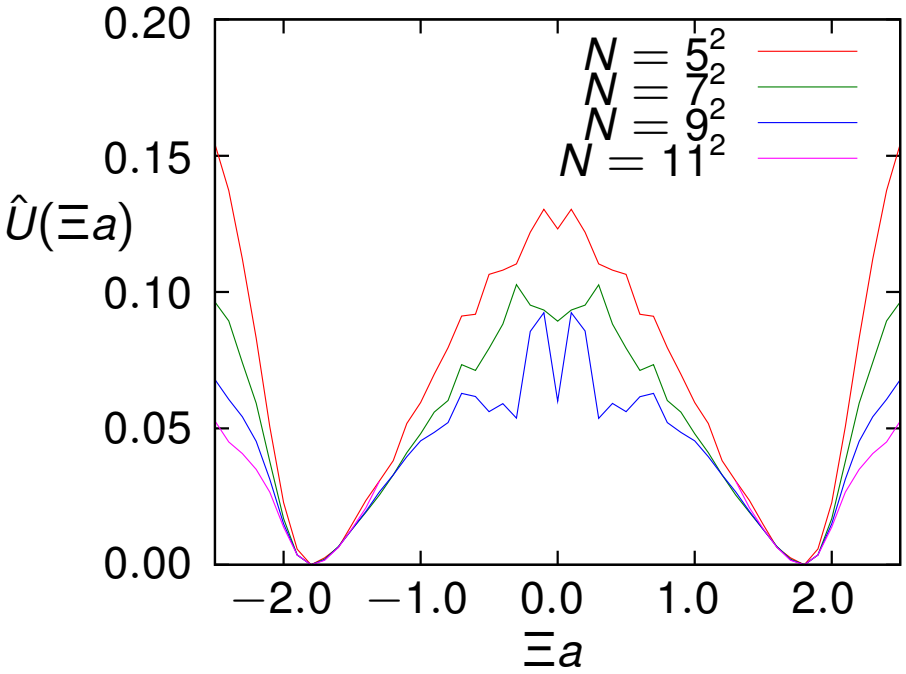
- discussed by P. Hasenfratz at Lattice '01
- solved by [BNW10], calculated logarithmic corrections

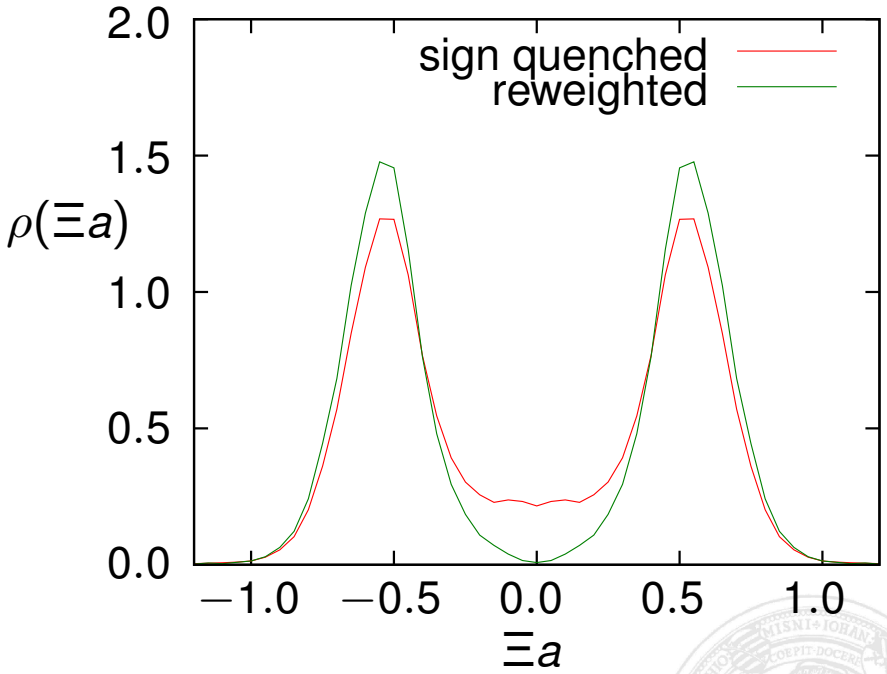
	N_s	u_0	$\sigma(2, u_0)$	deviation
Balog et. al.	16	1.0595	1.27244(2)	0.01123
	24	1.0595	1.26809(3)	0.00688
	32	1.0595	1.26594(4)	0.00473
	64	1.0595	1.26291(2)	0.00170
SLAC	20	1.0595	1.2641(7)	0.00295
	∞	1.0595	1.2604(13)	0.00081

- SLAC derivative shows much less lattice artifacts and improved continuum limit

Chiral Symmetry

using SLAC derivative

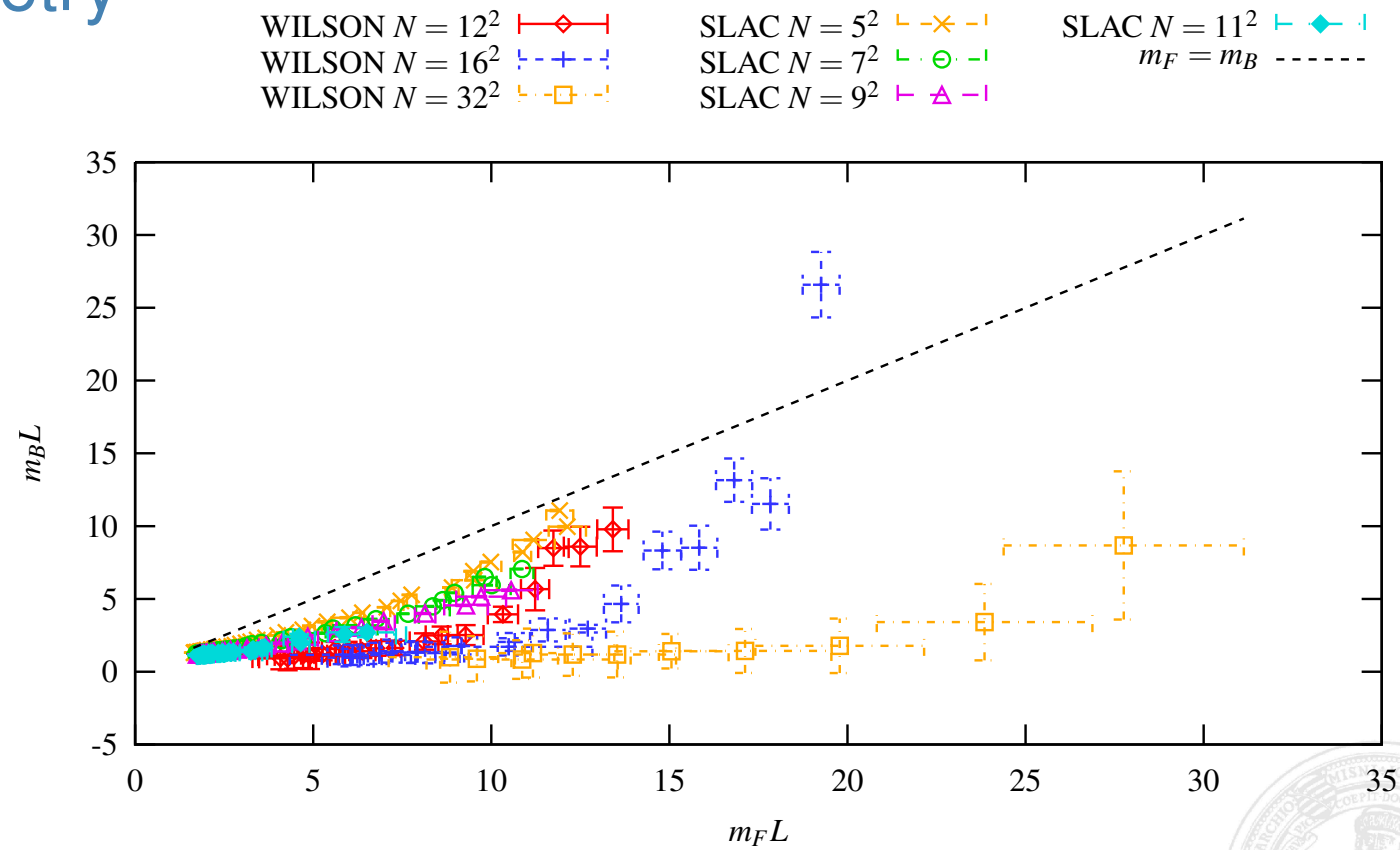




- no running of minima visible
- chiral symmetry spontaneously broken in the continuum limit
- negative sign configurations contribute primarily to $\Xi \approx 0$

Supersymmetry

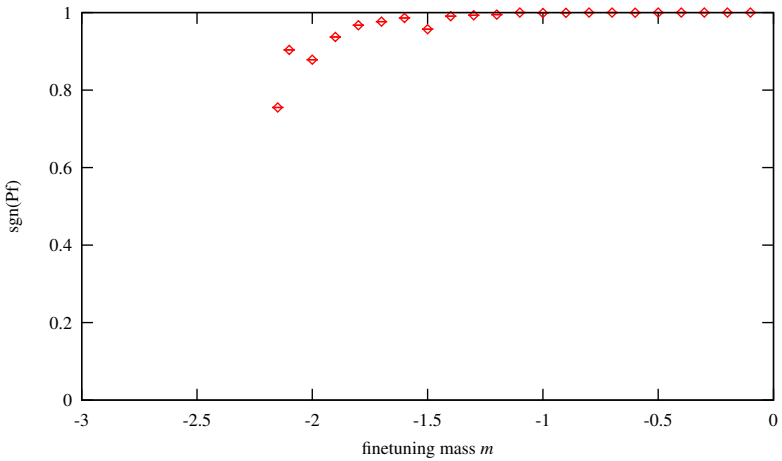
Masses

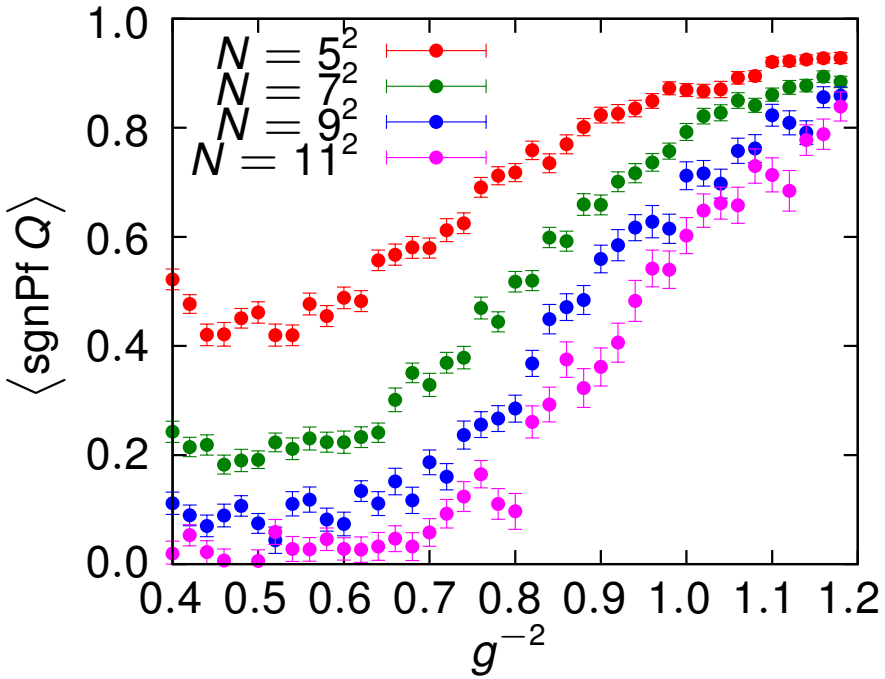


- $m_F L =$ fixed marks constant physics
- bosonic mass decreases with greater lattice volume and hints at broken supersymmetry in the continuum limit ☹

Algorithmic Improvements

Sign Problem





- sign problem dependent on finetuning mass, but still under control

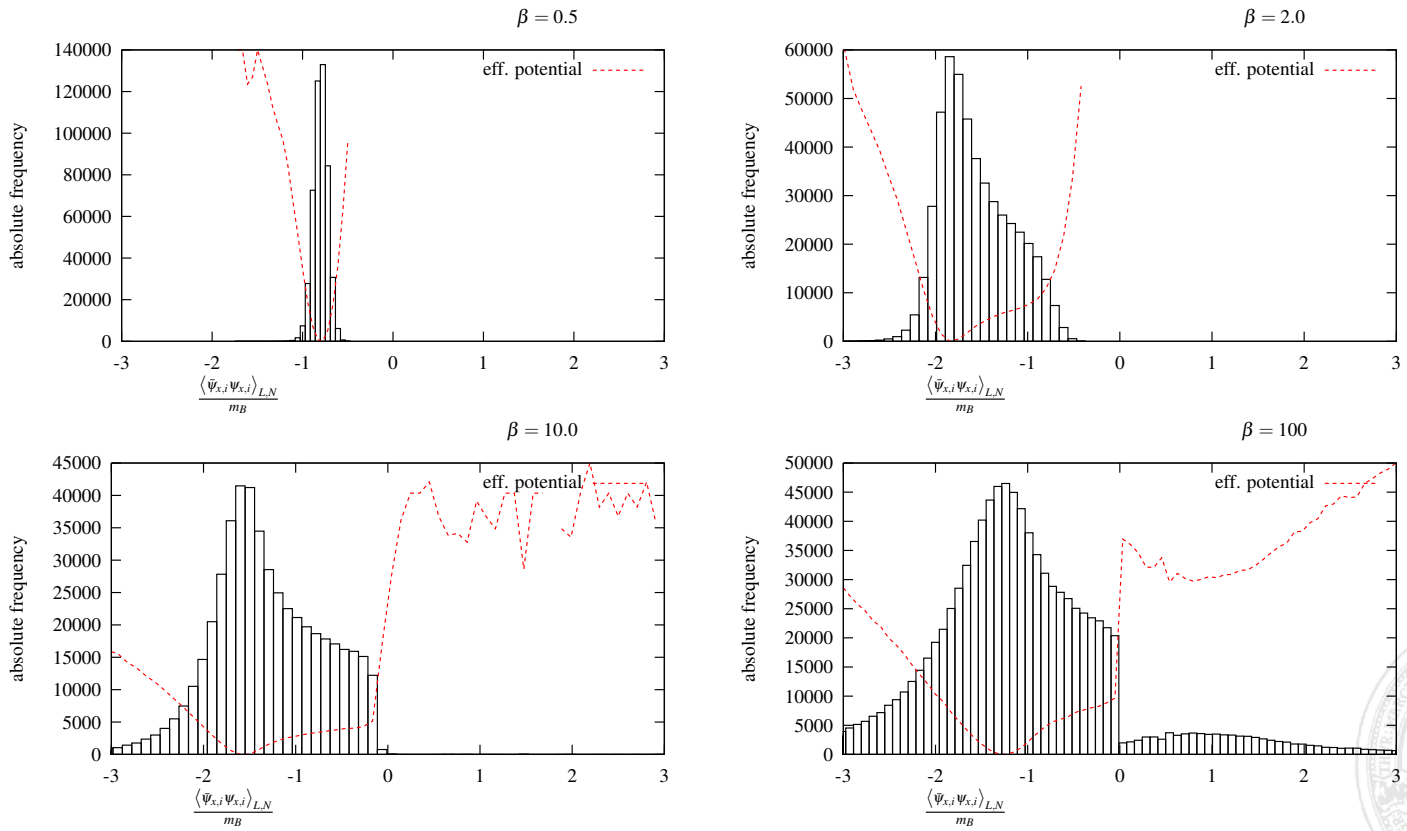
Condition Numbers

- factor 10 speedup achieved by even-odd preconditioning

	original	reduced	even-odd
condition number	$1.6(6) \cdot 10^{19}$	$1.3(6) \cdot 10^{8}$	$1.4(7) \cdot 10^{3}$
cg solver steps	454(10)	152(1)	48(1)

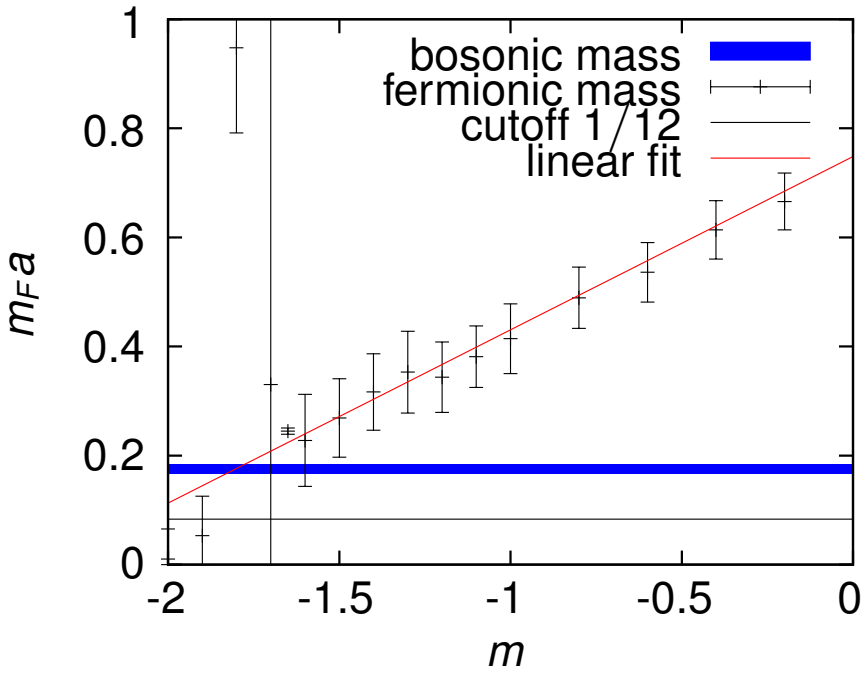
Chiral Symmetry

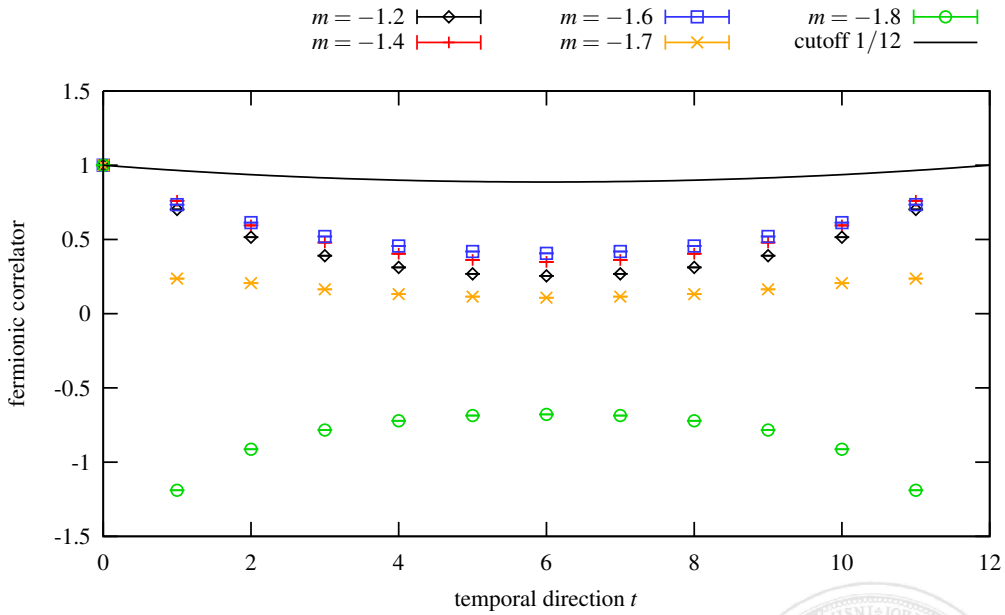
using Wilson derivative



- chiral condensate shows remnants of spontaneous breakdown for large g^{-2}!

Finetuned Ensemble





- finetuning towards smaller fermion masses possible
- exact matching at finite volume hindered by distorted correlator

Summary

- twisted susy formulation breaks O(3) symmetry in the continuum limit
- stereographic projection and group-valued fields provide exact O(3) symmetry on the lattice
- SLAC derivative reproduces exact result for step-scaling function to very high precision
- no degeneracy of bosonic and fermionic masses in the naive discretisation
- improved continuum limit by finetuning requires high performance (condition numbers, sign problem), provided by even-odd preconditioned Wilson fermions using RHMC
- finetuned simulations still running ...

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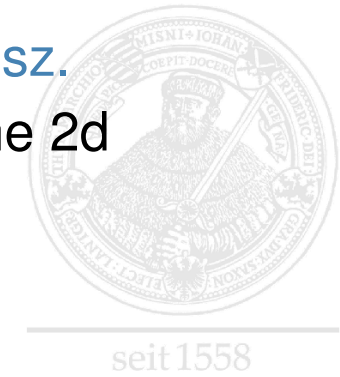

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