Relativistic Coupled-Channel Quark-Model Approach to Meson Resonances

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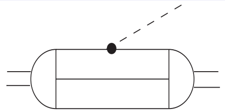
Outline

Introduction

- Resonance character of hadron excitations usually not taken into account in constituent-quark models
- Bound-state wave functions + elementary decay vertex



- Partial (hadronic) decay widths result in general too small

Strong decays of hadron resonances too small in RCQMs

- **Mesons**
  - Ricken, R., Koll, M., Merten, D.: *Strong two-body decays of light mesons.* Eur. Phys. J. A 18, 667 (2003)
- **Baryons**
  - Metsch, B., Löring, U., Merten, D., Petry, H.: *The spectrum and strong decays of baryons in a relativistic quark model.* Eur. Phys. J. A 18, 189 (2003)
  - Melde, T., Plessas, W., Wagenbrunn, R.F.: *Covariant calculation of mesonic baryon decays.* Phys. Rev. C 72, 015207 (2005); Erratum: Phys. Rev. C 74, 069901 (2006)
  - Sengl, B., Melde, T., Plessas, W.: *Covariant calculation of strange decays of baryon resonances.* Phys. Rev. D 76, 054008 (2007)

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Coupled-channel formalism

- Explicit coupling to decay channels
- Introduces additional meson/$q\bar{q}$ degrees of freedom
- Provides dressing of bound states and description of resonances with finite width
- Convenient starting point:

  RCQM with Goldstone-boson-exchange hyperfine interaction
  $\rightarrow$ provides direct coupling to pions

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Framework

Point form of Poincaré-invariant QM:

$$[P_\mu, P_\nu] = 0,$$
$$[J_{\mu\nu}, P_\kappa] = i(g_{\nu\kappa} P_\mu - g_{\mu\kappa} P_\nu),$$
$$[J_{\mu\nu}, J_{\kappa\lambda}] = -i(g_{\mu\kappa} J_{\nu\lambda} - g_{\nu\kappa} J_{\mu\lambda} + g_{\nu\lambda} J_{\mu\kappa} - g_{\mu\lambda} J_{\nu\kappa}).$$

- $P^\mu = (M, \vec{P})$ interaction-dependent
- Generators $J^{\mu\nu}$ of general Lorentz transformations kinematical
- Allows for manifestly covariant observables

Bakamjian-Thomas construction:

$$M := M_{free} + M_{int}$$
$$P^\mu = (M_{free} + M_{int}) V^\mu_{free}$$

Eigenvalue problem:

$$M |\psi\rangle = m |\psi\rangle$$

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Coupled-Channel mass operator

2-channel mass operator:

$$\begin{pmatrix} M_n & K^\dagger \\ K & M_{n+1} \end{pmatrix}$$

- n- and (n+1)-particle channels are coupled
- K . . . operator for emission/absorption of additional particle
  - n=1: e.g., dressing of a single bare particle by meson loops
  - n=2: e.g., dressing of a bound/bare two-particle state (meson) by meson loops
  - n=3: e.g., dressing of a bound/bare three-particle state (baryon) by meson loops

Eigenvalue equation for mass operator
(with additional π d.o.f.)

$$\begin{pmatrix} M_{cl} & K^\dagger \\ K & M_{cl,\pi} \end{pmatrix} \begin{pmatrix} |\psi_{q\bar{q}}\rangle \\ |\psi_{q\bar{q},\pi}\rangle \end{pmatrix} = m \begin{pmatrix} |\psi_{q\bar{q}}\rangle \\ |\psi_{q\bar{q},\pi}\rangle \end{pmatrix}$$

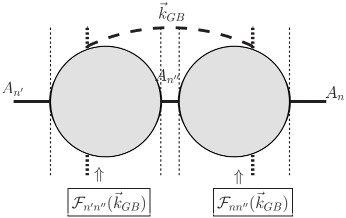
- $M_{cl} = M_{free} + M_{conf}\,, \qquad M_{cl,\pi} = M_{cl} + M_\pi$
- $K \ldots$ vertex interaction operator (quark coupling to π)
- $|\psi_{q\bar{q}}\rangle \ldots q\bar{q}$ component of mass-operator eigenstate
- $|\psi_{q\bar{q},\pi}\rangle \ldots q\bar{q}, \pi$ component of the mass-operator eigenstate
- **Feshbach reduction:**

$$\left[M_{cl} + K^\dagger(\mathbf{m} - M_{cl,\pi} + i\epsilon)^{-1} K\right] |\psi_{q\bar{q}}\rangle = \mathbf{m}\, |\psi_{q\bar{q}}\rangle$$

Expansion of dressed meson state $|\psi_{q\bar{q}}\rangle$

- Expand $|\psi_{q\bar{q}}\rangle$ in terms of eigenstates $|v, n\rangle$ of M_{cl}
- Expansion coefficients $\langle v, n \mid \psi_{q\bar{q}}\rangle = 2v_0\delta^3\left(\vec{v} - \vec{V}\right) A_n$ satisfy

$$\sum_{n'} \left(m_n \delta_{nn'} - V_{opt}^{nn'}(\mathbf{m}) \right) A_{n'} = \mathbf{m} A_n$$



$$V_{opt}^{nn'}(\mathbf{m}) = \sum_{n''} \int_0^\infty dk_{GB} \frac{\mathcal{I}(k_{GB})}{(\mathbf{m} - \omega_{n''} - \omega_{GB} + i\epsilon)}$$

Propagator Singularities

$$V_{opt}^{nn'}(\mathbf{m}) = \sum_{n''} \int_0^\infty dk_{GB} \frac{\mathcal{I}(k_{GB})}{(\mathbf{m} - \underbrace{\omega_{n''}}_{\sqrt{m_{n''}^2 + k_{GB}^2}} - \underbrace{\omega_{GB}}_{\sqrt{m_{GB}^2 + k_{GB}^2}} + i\epsilon)}$$

- $+i\epsilon$ ... Feynman prescription to avoid singularities for $\mathbf{m} > \omega_{n''} + \omega_{GB}$
- Imaginary part in optical potential

$$V_{opt}^{nn'}(\mathbf{m}) = \frac{1}{\mathcal{C}_1} \int_0^\infty dk_{GB} \frac{\mathcal{I}'(k_{GB})}{(k_{GB} - k_{GB}^0 - i\epsilon)} =$$

$$\frac{1}{\mathcal{C}_1} \mathcal{P}.\mathcal{V}. \int_0^\infty dk_{GB} \frac{\mathcal{I}'(k_{GB})}{(k_{GB} - k_{GB}^0)} + \frac{i\pi}{\mathcal{C}_1} \int_0^\infty dk_{GB} \mathcal{I}'(k_{GB}) \delta(k_{GB} - k_{GB}^0)$$

- Imaginary part in mass eigenvalues $\mathbf{m} = m_R + i m_I$
- Resonance decay width: $\Gamma = 2|m_I|$

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Parameters

- $\omega \to \rho\pi$ decay

Prefixed Parameters	Free Parameters

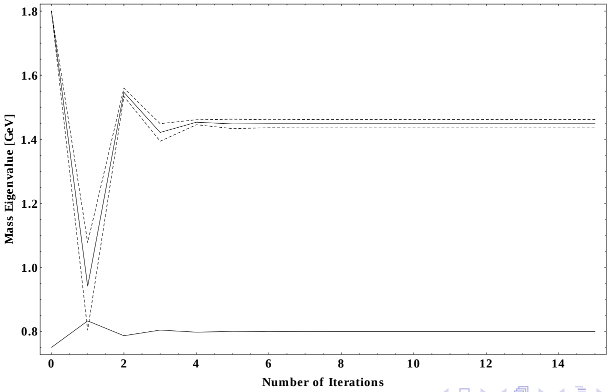
Quark masses	GB mass	Osc. Length	Shift param.
m_q	m_{GB}	a	V^0
0.34 [GeV]	0.135 [GeV]	0.28 [GeV]	0.1 [GeV^2]

- Harmonic oscillator confinement in M_{cl}^2
- The two free parameters are fixed such that the ground and the first excited state of the ω are approximately reproduced
- The πqq coupling constant in K taken from GBE RCQM
  - Restricted by Goldberger-Treiman relation: $0.67 \lesssim \frac{g^2}{4\pi} \lesssim 1.19$
- Simplifications: neglect spins and orbital excitations

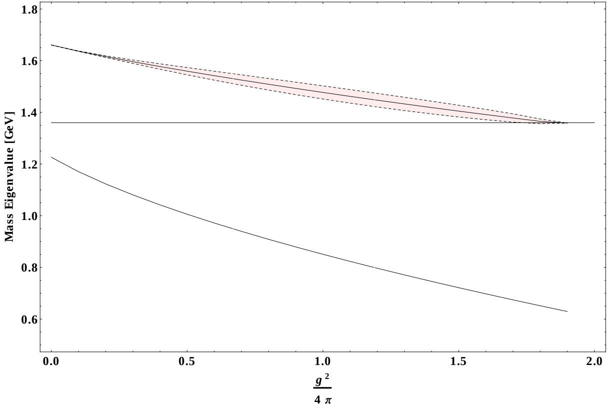
Self-Consistent Solution Method

- Solving the eigenvalue equation by iteration

$$\sum_{n'} \left(m_n \delta_{nn'} - V_{opt}^{nn'}(\mathbf{m}) \right) A_{n'} = \mathbf{m} A_n$$



Dependence on the coupling constant



- Decay width develops as coupling g to decay channel grows

Discussion

- Decay width vanishes when $Re[m_i > 0]$ approaches $m_0 + m_\pi$, where m_0 is the harmonic oscillator ground-state mass
- Calculated width Γ is not the true physical decay width for resonance $\Longrightarrow$ groundstate $+ \pi$
- Rather Γ describes the π-decay of the physical resonance to the bare component of the ground state

Further improvement:

Discussion

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Further improvement:

- Add instantaneous contribution from GBE to M_{cl}
- Expand $|\psi_{q\bar{q}}\rangle$ in terms of eigenstates of $(M_{cl} + \mathcal{V}_{hf})$ ($\widehat{=}$ redefinition of bare hadron)

Introduction

Framework

Application to mesons

Summary and outlook

Summary and Outlook

- We have paved the way to a relativistic coupled-channel quark-model description of hadrons
- More realistic
  - Dressing of ground states
  - Resonances with finite widths
- Simple scalar toy model shows already increase of decay widths
- Improve input for "bare hadron"
- Inclusion of explicit flavor and spin d.o.f. (ongoing ...) (calculate, e.g., $\omega\rho\pi$ vertex form factor)
- Extension to baryon resonances

Thank you ...

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