Deuteron Form Factors in Point-Form Relativistic Quantum Mechanics

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Structure of the talk

I. Point form of relativistic quantum mechanics

II. Calculation of EM currents and form factors

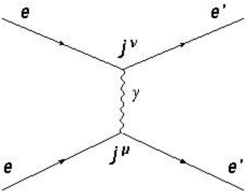
III. Current of spin-0 bound state

IV. Deuteron form factors

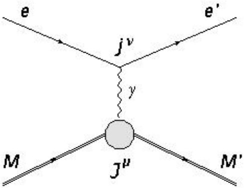
I. Point form of relativistic quantum mechanics

Motivation

Electromagnetic structure of few-body bound states probed in elastic electron scattering
$\Rightarrow$ Form factors (encode e.m. structure)



$\longrightarrow$



$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{point}} |F(Q^2)|^2$$

Motivation

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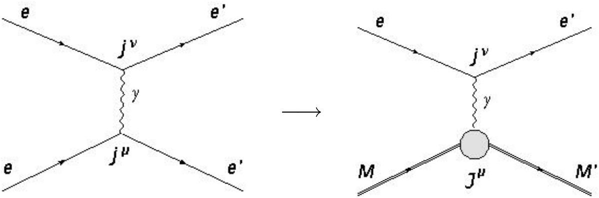
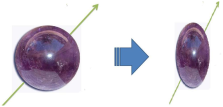


$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{point}} |F(Q^2)|^2$$

- FFs are Lorentz invariant coefficients in e.m. current J^μ
- J^μ → contains info about QED & QCD
  → consistent with relativity



Aim: formalism to describe e.m. currents of few-body bound states in terms of the constituents' currents

1. Poincaré invariance
2. Non-perturbative nature of strong interactions

Conditions for the current:

1. Lorentz covariance
2. Current conservation
3. Cluster separability (for relativistic QM)

Point form relativistic quantum mechanics

For any Poincaré (PC) invariant theory...

$$
\begin{aligned}
[P^0, \vec{P}] &= 0 & \qquad [P^i, P^j] &= 0 \\
[\vec{J}, P^0] &= 0 & \qquad [J^i, P^j] &= i\epsilon^{ijk} P^k \\
[J^i, J^j] &= i\epsilon^{ijk} J^k & \qquad [J^i, K^j] &= i\epsilon^{ijk} K^k \\
[\vec{K}, P^0] &= i\vec{P} & \qquad [K^i, P^j] &= i\delta^{ij} P^0 \\
[K^i, K^j] &= -i\epsilon^{ijk} J^k & &
\end{aligned}
$$

Interactions in $P^0 \Rightarrow$ interactions in either $\vec{K}$ or $\vec{P}$

Point form relativistic quantum mechanics

Dirac's forms of dynamics:

- Instant form → P^0, K^i contain interactions
- Point form → P^μ contain interactions
- Front form → $P^- = P^0 - P^3$, $F^1 = K^1 - J^1$, $F^2 = K^2 + J^1$ contain interactions

We use the **point form** of relativistic quantum mechanics:
$\hat{P}^\mu$ contain interactions, $\vec{J}$, $\vec{K}$ interaction free

$$\text{Bakamjian-Thomas} \Rightarrow \hat{P}^\mu = \hat{M}\hat{V}^\mu_{free}$$

Relativistic coupled channel approach

The Bakanjian-Thomas construction

- Coupled channel approach
- Dynamic of the exchanged particle is fully taken into account
- $|eq\bar{q}\rangle$ and $|eq\bar{q}\gamma\rangle$ coupled by a eigenvalue equation

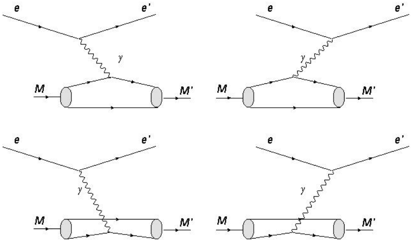
$$\begin{pmatrix} \hat{M}^{conf}_{eq\bar{q}} & \hat{K} \\ \hat{K}^{\dagger} & \hat{M}^{conf}_{eq\bar{q}\gamma} \end{pmatrix} \begin{pmatrix} |\psi_{eq\bar{q}}\rangle \\ |\psi_{eq\bar{q}\gamma}\rangle \end{pmatrix} = m \begin{pmatrix} |\psi_{eq\bar{q}}\rangle \\ |\psi_{eq\bar{q}\gamma}\rangle \end{pmatrix}$$

$\hat{K} \rightarrow$ depends on $\hat{P}^{\mu} = \hat{M}\hat{V}^{\mu}_{free}$, depends on $\mathcal{L}^{int}_{QED}$

Optical potential derivation by Feshbach reduction

$$\begin{pmatrix} \hat{M}^{conf}_{eq\bar{q}} & \hat{K} \\ \hat{K}^{\dagger} & \hat{M}^{conf}_{eq\bar{q}\gamma} \end{pmatrix} \begin{pmatrix} |\psi_{eq\bar{q}}\rangle \\ |\psi_{eq\bar{q}\gamma}\rangle \end{pmatrix} = m \begin{pmatrix} |\psi_{eq\bar{q}}\rangle \\ |\psi_{eq\bar{q}\gamma}\rangle \end{pmatrix}$$

$$\longrightarrow (\hat{M}_{eq\bar{q}} - m)|\psi_{eq\bar{q}}\rangle = \underbrace{\hat{K}^{\dagger}(\hat{M}_{eq\bar{q}\gamma} - m)^{-1}\hat{K}}_{\hat{V}_{opt}(m)}|\psi_{eq\bar{q}}\rangle$$

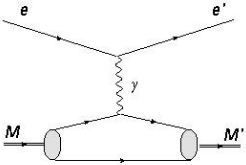


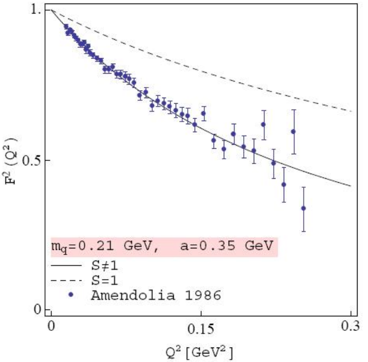
$$\langle v'; \vec{k}'_e, \mu'_e; \vec{k}'_C, n|\hat{V}^{\text{const}}_{\text{opt}}(m)|v; \vec{k}_e, \mu_e \vec{k}_C, n\rangle \quad \propto \quad j^{\mu}_e \ J^{\text{micro}}_{\mu}$$

Examples: pseudoscalar mesons: $\pi^{\pm}$ form-factor

[Elmar P.Biernat, W.Schweiger, K.Fuchsberger, W.Klink, Phys.Rev C **79**, 055203 (2009)]

$$J^{\mu}(\vec{k}'_C,\vec{k}_C)=\sum_{\mu'_q\mu_q}\int d^3\tilde{k}'_q\ldots j^{\mu}_{\mu'_q\mu_q}(\vec{k}'_q,\vec{k}_q)\;\mathcal{S}\;\Psi(\tilde{k}'_q,\mu'_q)\Psi(\tilde{k}_q,\mu_q)$$





Properties of J^{μ}

- Lorentz covariance
- Current conservation
- PROBLEM: Cluster separability

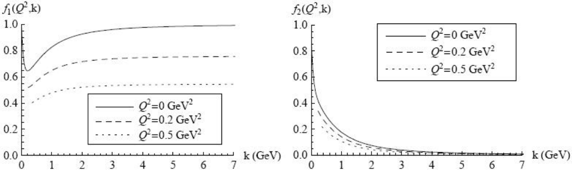
Problem: Cluster separability violation

The limit $k \to \infty$

Bakamjian-Thomas construction ⇒ cluster properties are violated

$$\boxed{\Rightarrow\ J^\mu(\vec{k}'_C, \vec{k}_C) = f_1(Q^2, s)(k'_C + k_C)^\mu + f_2(Q^2, s)(k'_e + k_e)^\mu}$$

E.P. Biernat & W. Schweiger



$$\xcancel{J^\mu(k'_C, k_C) = F(Q^2)(k'_C + k_C)^\mu}$$

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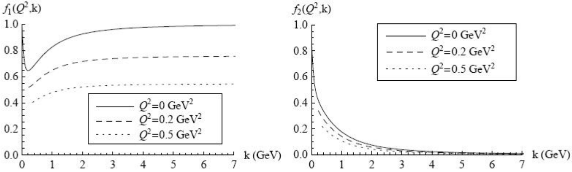


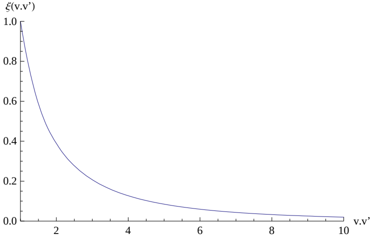
$$\xcancel{J^\mu(k'_C, k_C) = F(Q^2)(k'_C + k_C)^\mu}$$

For $|\vec{k}_C| \to \infty$,

- $f_1(Q^2, s) \to F(Q^2)$, and $f_2(Q^2, s) \to 0$: Cluster properties are restored
- Usual structure recovered $J^\mu(k'_C, k_C) = F(Q^2)(k'_C + k_C)^\mu$
- Equivalence with standard front form result for J^+ in $q^+ = 0$ frame
  [Chung, Coester, Polyzou; PLB 205, 1988]

Heavy-light systems $m_Q \to \infty$

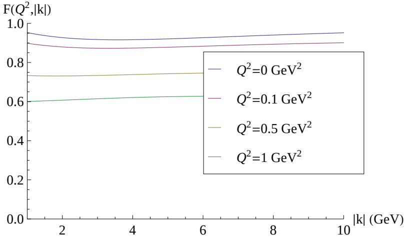
[M.G.R. , W. Schweiger, paper in preparation]

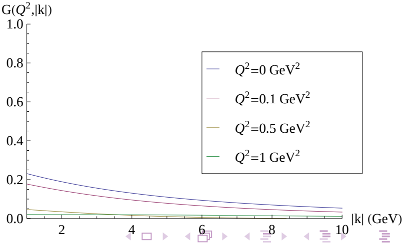


Cluster problem vanishes also when one of the quark is much heavier than the other

$a = 0.55GeV$

B





IV. Deuteron form factors

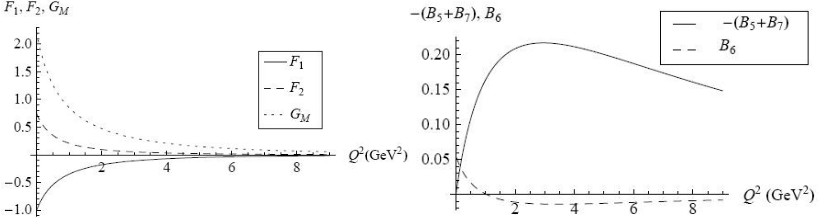
Spin-1 systems

- Extend the definition of the current to spin-1 mesons
- Consider the most general combination of linear independent covariants including $K_e := k'_e + k_e$ → 11 covariants

$$
\begin{aligned}
&J^\mu(k_C,\mu_C;k'_C,\mu'_C;K_e) = \\
&= \left[f_1 \epsilon'^* \cdot \epsilon + f_2 \frac{(\epsilon'^* \cdot q)(\epsilon^* \cdot q)}{2m_C^2} \right] K_C^\mu + g_M \left[\epsilon'^{*\mu}(\epsilon \cdot q) - \epsilon^\mu(\epsilon'^* \cdot q) \right] \\
&+ \frac{m_C^2}{2K_e \cdot k_C} \Big[b_1 (\epsilon'^* \cdot \epsilon) + b_2 \frac{(q \cdot \epsilon'^*)(q \cdot \epsilon^*)}{m_C^2} + b_3 m_C^2 \frac{(K_e \cdot \epsilon'^*)(K_e \cdot \epsilon^*)}{(K_e \cdot k_C)^2} \\
&+ b_4 \frac{(q \cdot \epsilon'^*)(K_e \cdot \epsilon) - (q \cdot \epsilon)(K_e \cdot \epsilon'^*)}{2(K_e \cdot k_C)} \Big] K_e^\mu \\
&+ \left[b_5 m_C^2 \frac{(K_e \cdot \epsilon'^*)(K_e \cdot \epsilon)}{(K_e \cdot k_C)^2} + b_6 \frac{(q \cdot \epsilon'^*)(K_e \cdot \epsilon) - (q \cdot \epsilon)(K_e \cdot \epsilon'^*)}{2K_e \cdot k_C} \right] K_C^\mu \\
&+ b_7 m_C^2 \frac{\epsilon'^{*\mu}(\epsilon \cdot K_e) + \epsilon^\mu(\epsilon'^* \cdot K_e)}{K_e \cdot k_C} + b_8 q^\mu \frac{(q \cdot \epsilon'^*)(K_e \cdot \epsilon) + (q \cdot \epsilon)(K_e \cdot \epsilon'^*)}{2K_e \cdot k_C}
\end{aligned}
$$

Spin-1 systems

[E. P. Biernat, PhD Thesis], ρ-meson



- limit $k \to \infty$ does not remove spurious form factors $B_5(Q^2)$, $B_6(Q^2)$, $B_7(Q^2)$ and $B_8(Q^2)$
  - B_7, B_8: violation of current conservation
  - $(B_5 + B_7)$: violation of the so called angular condition

$$(1+2\eta)J^0_{11} + J^0_{1-1} - 2\sqrt{2\eta}J^0_{10} - J^0_{00} = -(B_5 + B_7) \neq 0$$

- BUT: physical form factors $F_1(Q^2)$, $F_2(Q^2)$, $G_M(Q^2)$ can be uniquely extracted from *good* matrix elements J^0_{11}, J^0_{1-1}, J^2_{11}
- Resemblance to covariant light-front approach of Karmanov *et al.* [Phys.Rept.300, 1998]

Deuteron form factors

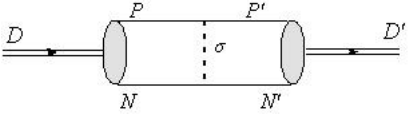
Benchmark calculation: The Walecka Model

Investigate the simplest (strongly) bound two-body state within a common model

$$\mathcal{L}_{int} = g_\sigma \bar{\psi}\psi\phi + i g_\omega \bar{\psi}\gamma_\mu \psi \chi^\mu + i g_\sigma \bar{\psi}\psi\eta + g_\eta \bar{\psi}\gamma_\mu \psi \theta^\mu$$

Parameters computed by Elmar P. Biernat for an instantaneous potential. Form factors already obtained for this static potential

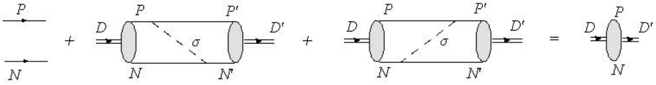
$$\begin{aligned} m_\sigma &= 400 \text{ MeV} \\ g_\sigma^2/4\pi &= 6.31 \\ m_\omega &= 782.7 \text{ MeV} \\ g_\omega^2/4\pi &= 18.617 \end{aligned}$$

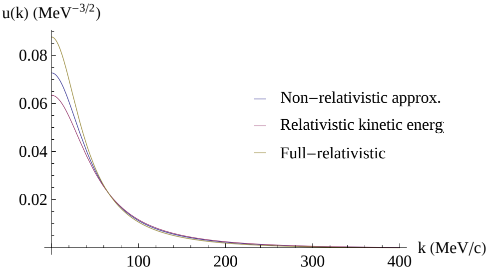


Starting point to investigate relativistic effects in different approaches

The relativistic wave function

$$\left(\hat{M}_0 + \hat{V}(m)\right)|\psi\rangle = \left(\hat{M}_{NP} + \hat{K}_\sigma(m - \hat{M}_{NP\sigma})^{-1}\hat{K}_\sigma^\dagger\right)|\psi\rangle = m|\psi\rangle$$

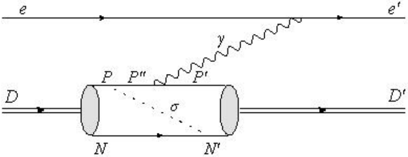




Deuteron form factors

Benchmark calculation: The Walecka Model

The next task...



- Investigate relativistic effects: retardation effects
- One-particle exchange potential
- Relativistic wave function

4-channel approach

$$\begin{pmatrix} \hat{M}_{eNP} & \hat{K}_\gamma & \hat{K}_\sigma & 0 \\ \hat{K}_\gamma^\dagger & \hat{M}_{eNP\gamma} & 0 & \hat{K}_\sigma \\ \hat{K}_\sigma^\dagger & 0 & \hat{M}_{eNP\sigma} & \hat{K}_\gamma \\ 0 & \hat{K}_\sigma^\dagger & \hat{K}_\gamma^\dagger & \hat{M}_{eNP\gamma\sigma} \end{pmatrix} \begin{pmatrix} |\psi_{eNP}\rangle \\ |\psi_{eNP\gamma}\rangle \\ |\psi_{eNP\sigma}\rangle \\ |\psi_{eNP\gamma\sigma}\rangle \end{pmatrix} = m \begin{pmatrix} |\psi_{eNP}\rangle \\ |\psi_{eNP\gamma}\rangle \\ |\psi_{eNP\sigma}\rangle \\ |\psi_{eNP\gamma\sigma}\rangle \end{pmatrix}$$

Deuteron form factors

Exchange current contributions

The optical potential...

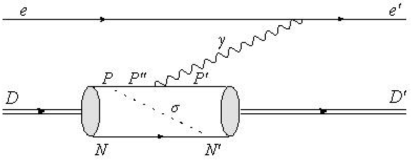
$$
\begin{array}{ll}
 & \{\hat{K}_\sigma (m - \hat{M}_{eNP\sigma})^{-1} \hat{K}_\sigma^\dagger + \\
+ & \hat{K}_\gamma (m - \hat{M}_{eNP\gamma} - \hat{K}_\sigma (m - \hat{M}_{eNP\sigma\gamma})^{-1} \hat{K}_\sigma^\dagger)^{-1} \hat{K}_\gamma^\dagger \\
+ & \hat{K}_\gamma (m - \hat{M}_{eNP\gamma} - \hat{K}_\sigma (m - \hat{M}_{eNP\sigma\gamma})^{-1} \hat{K}_\sigma^\dagger)^{-1} \times \\
 & \times \hat{K}_\sigma (m - \hat{M}_{eNP\sigma\gamma})^{-1} \hat{K}_\gamma^\dagger (m - \hat{M}_{eNP\sigma})^{-1} \hat{K}_\sigma^\dagger + \\
+ & \hat{K}_\sigma (m - \hat{M}_{eNP\sigma})^{-1} \hat{K}_\gamma (m - \hat{M}_{eNP\sigma\gamma})^{-1} \hat{K}_\gamma^\dagger (m - \hat{M}_{eNP\sigma})^{-1} \hat{K}_\sigma^\dagger + \\
+ & \hat{K}_\sigma (m - \hat{M}_{eNP\sigma})^{-1} \hat{K}_\gamma (m - \hat{M}_{eNP\sigma\gamma})^{-1} \hat{K}_\sigma^\dagger \times \\
 & \times (m - \hat{M}_{eNP\gamma} - \hat{K}_\sigma (m - \hat{M}_{eNP\sigma\gamma})^{-1} \hat{K}_\sigma^\dagger)^{-1} \hat{K}_\gamma^\dagger + \\
+ & \hat{K}_\sigma (m - \hat{M}_{eNP\sigma})^{-1} \hat{K}_\gamma (m - \hat{M}_{eNP\sigma\gamma})^{-1} \hat{K}_\sigma^\dagger \times \\
 & \times (m - \hat{M}_{eNP\sigma} - \hat{K}_\sigma (m - \hat{M}_{eNP\sigma\gamma})^{-1} \hat{K}_\sigma^\dagger)^{-1} \times \\
 & \times \hat{K}_\sigma (m - \hat{M}_{eNP\sigma\gamma})^{-1} \hat{K}_\gamma^\dagger (m - \hat{M}_{eNP\sigma})^{-1} \hat{K}_\sigma^\dagger \} |eNP\rangle = (m - \hat{M}_{eNP}) |eNP\rangle
\end{array}
$$

Deuteron form factors

Exchange current contributions

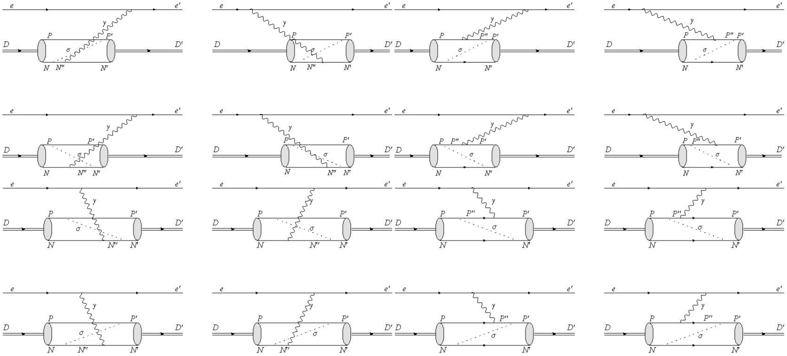
Examples...

$$\hat{K}_\sigma (m - \hat{M}_{eNP\sigma})^{-1} \hat{K}_\gamma (m - \hat{M}_{eNP\sigma\gamma})^{-1} \hat{K}_\sigma^\dagger \times$$
$$\times (m - \hat{M}_{eNP\gamma} - \hat{K}_\sigma (m - \hat{M}_{eNP\sigma\gamma})^{-1} \hat{K}_\sigma^\dagger)^{-1} \hat{K}_\gamma^\dagger$$



Deuteron form factors.

Exchange current contributions



...form factors are being computed at this moment

Summary

- Relativistic formalism to derive current and form factors of bound few-body systems consistent with the binding forces
- Cluster problems which appear because of the BT construction remain under control → spurious form factors.
- Physical form factors can be uniquely extracted from certain matrix elements of the current
- Analytical and numerical agreement with front form of dynamics
- Study relativistic effects in few-body systems

Thank you!