MCRG Flow for the Nonlinear Sigma Model

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So you want to quantize gravity ...

RG Approach to QFT

- all information stored in correlation functions

\[ \langle \tilde{\phi}(x_0) \ldots \tilde{\phi}(x_n) \rangle = \mathcal{N} \int \mathcal{D}\phi \phi(x_0) \ldots \phi(x_n) e^{-S[\phi]} \]

- perturbatively calculated correlation functions may contain divergences

- **Regularization**: render theory finite by some cutoff \( \Lambda \)

- **Renormalization**: introduce counter terms such that for \( \Lambda \rightarrow \infty \) the theory remains finite

- perturbative renormalizability strong guideline for models
So you want to quantize gravity ...

Effective vs. Fundamental Theories

- theory at scale $k$ described by effective average action $\Gamma_k$
- upper (ultraviolet) cutoff $\Lambda$: $\Gamma_k = S_{mic}$
- use RG flow to integrate out fluctuation until lower cutoff $k = \lambda$
- fundamental theory valid on all scales, limit $\Lambda \to \infty$ and $\lambda \to 0$ exists
Setting the Stage

MCRG Method

Results

Summary

So you want to quantize gravity ...

Asymptotic Safety

- unified theory requires quantization of spacetime metric
- perturbative approach leads to severe divergences
- theory not renormalizable in perturbative way

- Asymptotic Safety Scenario (Weinberg 1980): gravity nonperturbatively renormalizable
- needs nongaussian (ultraviolet) fixed point (at least one coupling different from zero)
- asymptotic freedom (QCD): theory approaches gaussian fixed point for $k \to \infty$
Setting the stage

- nontrivial UV fixed point suspected in \( D > 2 \) nonlinear sigma models by perturbation theory and functional renormalization group methods

\[ D = 3 \ \text{O}(N) \ \text{NLSM} \]

\[ S = \frac{1}{2g^2} \int d^3 x \ \partial_\mu \vec{\phi} \partial^\mu \vec{\phi}, \ \text{where} \ \vec{\phi}^2 = 1 \]

- Flow diagram determinable from the lattice via Monte Carlo Renormalization Group (MCRG) techniques
RG picture on the lattice

- (discrete) lattice momenta spectrum cut off by lattice spacing \( a \) and box volume \( V = aN \)
- lattice simulation equivalent to integrating out all fluctuations inbetween
- correlation functions determined directly

Blocking spin transformation: \( a \to 2a, N \to N/2 \)
Setting the Stage

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for every configuration ...

- use blockspin transformation to integrate out fluctuations
- determine renormalized couplings via demon method

Demon Method (Creutz 1984)

- idea: couple "demon" systems to original configuration and simulate joint partition function
- measured demon energies allow to extract renormalized couplings

Observables

- (discrete) beta function \( \tilde{\beta}_i = \tilde{\beta}(g_i) = -(g_i^{\text{blocked}} - g_i) \)
- (discrete) stability matrix \( S_{ij} = \frac{\partial \tilde{\beta}_i}{\partial g_j} \)
Systematic Errors

- finite volume errors and discretization errors can be controlled
- effective action used in demon method leads to truncation errors
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- finite volume errors and discretization errors can be controlled
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- half group property of RG transformation $R_s$ (blockspin + demon) violated

\[ R_s \circ R_s \neq R_{2s} \]

- idea: improved Blockspin-transformation (Anna Hasenfratz)

\[ \Phi_{\tilde{x}} \propto e^{C \cdot \Phi_{\tilde{x}} \sum_{x \in \Lambda_{\tilde{x}}} \phi_x} \]

- $C$ used to optimize transformation
- parametrize $C = \sum_i c_i g_i$ such that for $g \to \infty$, $C \to \infty$ holds
Fixed Points

- Monte Carlo literature: $\nu^{-1} = 1.4029(28)$
1-coupling case

- Hasenfratz: $C = cg$

![Graph showing data for different values of $g$.]
1-coupling case

- rescaled: $C = c'g^2$, with $c' \approx 17.8$
3-coupling case

only irrelevant coupling is added
3-coupling case

"only irrelevant coupling is added"
Summary

- systematic errors of demon method controlled efficiently by improved RG transformation
- flow diagram reveals two trivial IR fixed points (absolute order / chaos) and one nontrivial UV fixed point
- fixed point structure stable against truncation (only one relevant direction)
- finite N critical exponents compatible with large N expansion / high precision MC results
- outlook: determine constants for improved RG transformation, obtain critical exponents with better precision