Mesons in a Bethe-Salpeter approach beyond Rainbow-Ladder

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Outline of the research

- Investigate hadronic bound states (specifically, mesons) in QCD.
- Calculate:
  1. Mass spectrum
  2. Decay constants
  3. Form factors
  4. . .
- Bound states $\rightarrow$ Non-perturbative tools
- Chiral limit $\rightarrow$ heavy quarks
  $\Rightarrow$ Dyson-Schwinger and Bethe-Salpeter equations (DS/BS approach).
Dyson-Schwinger equations

- DSEs - infinite tower of coupled integral equations.
- Exact, non-perturbative, continuous formulation of a QFT.
- Relate the $n$-point Green’s functions to one another.
- Collapse tower by specifying higher GFs. 😐
- Don’t know exactly any higher GFs. 😞

⇒ Some truncation scheme required.
Mesons (q̅q bound states) described within DS/BS approach

First investigations concerned properties of light mesons (π, K, ρ...)

Only recently baryons within the three-body Faddeev equation
Mesons in a Bethe-Salpeter approach

- BSE formulated as an eigenvalue equation, with eigenvector $\Gamma(p, P)$, the quark-meson amplitude.
- Discrete eigenvalues are $P^2$, the meson masses.

$$\Gamma^M_{ab}(q, P) = \int K(k, q, P)S(k^+)\Gamma^M_{ab}(k, P)S(k^-)$$ (1)

The Bethe-Salpeter equation.
Fundamental objects in a BSE

Dressed quark propagator

\[ S(p) = \frac{i\gamma + M(p^2)}{p^2 + M^2(p^2)} Z_f(p^2) \]

Dressed gluon propagator

\[ D_{\mu\nu}(p) = (\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}) \frac{Z(p^2)}{p^2} \]

Dressed quark-gluon vertex

\[ \Gamma_\nu(p). \]

Each object satisfies a corresponding DSE.
The quark DSE

Usually one solves only a DSE for $S(p)$, using model Ansätze for $D_{\mu\nu}(p)$ and $\Gamma_{\nu}(p)$.

$$S^{-1}(p) = S_0^{-1}(p) + \int_q \gamma_{\mu} S(q) \Gamma_{\nu}(q,p) D_{\mu\nu}(p - q)$$ (2)

The quark Dyson-Schwinger equation.
Chiral symmetry and its contraints

- Truncations of the DSEs should respect symmetries.
- An important symmetry is the chiral symmetry.
- Preservation of the chiral symmetry ensures that
  1. The pion is massless in the chiral limit.
  2. Explicit $\chi_{SB}$ gives rise to the physical spectrum of mesons.
  3. Relations like the GMOR equation are satisfied.

$$f_\pi^2 m_\pi^2 = 2m_q \langle q\bar{q} \rangle$$  \hspace{1cm} (3)
The axWTI

- Constraints of the chiral symmetry expressed in the axial vector Ward-Takahashi identity.
- axWTI connects the quark-gluon vertex to the kernel of the BSE.

\[ -i P_\mu \Gamma^5_\mu = S_F^{-1}(p_+)\gamma_5 + \gamma_5 S_F^{-1}(p_-) - 2m_R \Gamma^5(p; P) \] (4)

The diagrammatic representation of axWTI
The Rainbow-Ladder Truncation

- Rainbow approximation in the quark DSE:
  \[ \Gamma_\nu(q,p) \rightarrow \gamma_\nu \]  
  (5)

- The ladder approximation in the meson BSE:
  \[ K(p,q; P) \rightarrow -G_{eff}(k^2) D_{\mu\nu}(k) \left( \frac{\lambda^a}{2} \gamma_\mu \right) \otimes \left( \frac{\lambda^a}{2} \gamma_\mu \right) \]  
  (6)

Models for \( G_{eff}(k) \):  
- R. Alkofer, C. Fischer, R. Williams, EPJ **A38** (2008)

The Rainbow-Ladder approximation
Why Rainbow-Ladder?

- An exceedingly simple truncation procedure.
- Preserves the axWTI.
- Satisfactory results for pseudoscalar and vector mesons, and (to some degree) for tensor mesons.
Bottomonium ground states in RL

The bottomonium ground state spectrum

M. Blank, A. Krassnigg, Phys Rev D, 84, 096014 (2011)
The isovector ground states in RL

M. Blank, A. Krassnigg, Phys Rev D, 84, 096014 (2011)

The isovector ground state spectrum
Shortcomings of RL

- Induces a $\gamma \otimes \gamma$ (vector-vector) coupling, a strong $L.S$ component with a wrong sign.
  $\Rightarrow$ RL is too attractive for some meson channels (i.e. scalar and pseudovector mesons).
- Well-suited only for pure $q\bar{q}$ states (flavour non-singlet).
  $\Rightarrow$ Can’t describe any phenomenon associated with flavour mixing (e.g. no decay channels).
  $\Rightarrow$ Improve method: use input from gluon and quark-gluon DSE, not Ansätze.
- Go Beyond the Rainbow.
The quark-gluon vertex

- Full quark-gluon vertex can be written as

\[ \Gamma_\nu(k, p) = Z_1 f \gamma_\nu + \Lambda_\nu \quad (7) \]

- RL replaces this with

\[ \Gamma(k, p) \to (Z_1 f + \Lambda(k^2)) \gamma_\nu, \quad k - \text{gluon momentum} \quad (8) \]

- Most interesting physics lies in the \( \Lambda_\nu \) part.
- Extract this physics with the help of the quark-gluon DSE.
The DSE for $\Gamma_\nu$

The full (a) and truncated (b) DSE for quark-gluon vertex.

e.g. R. Williams, arXiv:0912.3494v1
The BSE beyond RL

The a\times WTI preserving BSE corresponding to the truncated vertex DSE.
Results

Meson masses calculated using the rainbow-ladder (RL) and additional contributions from Abelian (AB), non-Abelian (NA) and pion back-reaction (PI) diagrams.

<table>
<thead>
<tr>
<th>Model</th>
<th>$m_\pi$</th>
<th>$m_\sigma$</th>
<th>$m_\rho$</th>
<th>$m_{a1}$</th>
<th>$m_{b1}$</th>
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<tr>
<td>RL</td>
<td>138</td>
<td>645</td>
<td>758</td>
<td>926</td>
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<tr>
<td>NA</td>
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<td>889</td>
<td>915</td>
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<tr>
<td>AB + NA</td>
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<td>883</td>
<td>878</td>
<td>1052</td>
<td>972</td>
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<tr>
<td>NA + PI</td>
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<td>820</td>
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<td>Experiment</td>
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<td>400-1200</td>
<td>776</td>
<td>1230</td>
<td>1230</td>
</tr>
</tbody>
</table>

R.Williams, arXiv:0912.3494v1
A note of caution

- Simple model beyond RL: keep the AB contribution to $\Gamma_\nu(p)$.
- 1st step: Replace gluon with $\delta$ function (Munczek-Nemirovsky truncation).
  Qualitative prediction: AB diagram repulsive.
- 2nd step: Replace vertex dressing gluon with $\delta$, keep momentum dependent gluon in quark DSE.
  Qualitative prediction: AB diagram attractive.
- 3rd step: Keep momentum dependent gluon everywhere.
  Qualitative prediction: AB diagram attractive.
  R. Williams, arXiv:0912.3494v1

Caution-making too many simplifications changes the general trend. Not good if predictions for BRL based on old results!
Conclusions and outlook

- RL the simplest truncation to preserve axWTI.
- Satisfactory description of pseudoscalar, vector and tensor mesons.
- Unsatisfactory description of scalar and axial-vector mesons, no flavour mixing, no decay channels.
- Include more physics-use input from the quark-gluon vertex DSE.
- Application to QCD-like theories?